6ᵗʰ International Symposium on Hydraulic Structures

Jun 28th, 10:30 AM

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Recommended Citation

Valero, D., Bung, D., Crookston, B., Matos, J. (2016). Numerical investigation of USBR type III stilling basin performance downstream of smooth and stepped spillways. In B. Crookston & B. Tullis (Eds.), Hydraulic Structures and Water System Management. 6th IAHR International Symposium on Hydraulic Structures, Portland, OR, 27-30 June (pp. x-x). doi:10.15142/T340628160853 (ISBN 978-1-884575-75-4).

6th International Symposium on Hydraulic Structures Portland, Oregon, USA, 27-30 June 2016
Hydraulic Structures and Water System Management
ISBN 978-1-884575-75-4 DOI: UUUUUUUUUU

Numerical Investigation of USBR Type III Stilling Basin Performance Downstream of Smooth and Stepped Spillways

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ABSTRACT

Recent systematic studies on air-water flows have included stepped spillways. However, to date, little has been investigated about how the hydraulic conditions on the stepped spillway may affect the design of traditional energy dissipation structures. In this paper, both smooth chute and stepped chute configurations terminating with the USBR type III stilling basin are tested by means of numerical modelling, allowing a qualitative comparison. Unsteady RANS equations have been employed together with VOF and RNG k-ε for free surface tracking and turbulence modeling, respectively. Eight different Froude numbers (F) ranging from 3.1 to 9.5 have been analyzed for a type III basin designed for F = 8, following recent studies conducted in a physical model by Reclamation. The basin flow structure is discussed for both smooth chute and stepped chute cases. Additionally, the modelled basin has been tested for design and adverse hydraulic conditions, obtaining a detailed insight of the role of each basin element and their adapting roles when insufficient tail water conditions exist.

Keywords: *Stepped chute, RANS, hydraulic jump energy dissipators, CFD, velocity decay, USBR type III stilling basin.*

1. INTRODUCTION

A stilling basin is a hydraulic structure located between the outlet works and the tail-water, intended to return flows safely to the downstream channel by dissipating excess kinetic energy (Vischer and Hager 1998, Chanson 2015). Its performance is directly linked to the inlet flow characteristics, which are influenced by the spillway geometry. In classic studies, a smooth invert chute has been considered ideal (Peterka, 1978); however, recent knowledge on stepped spillways (Chanson 2002, Amador et al. 2009, Meireles and Matos 2009, Bung 2011, Felder and Chanson 2013, Hunt et al. 2014, Matos and Meireles 2014) has encouraged practitioners to consider this type of hydraulic structure. A smooth chute provides little energy dissipation, therefore requiring concentrated energy dissipation in the stilling basin; whereas the stepped spillway works as a continuous dissipater, thus potentially reducing required stilling basin dimensions while also improving flow aeration conditions.

When a stepped spillway is considered, the stilling basin's chute blocks may be dispensable, thus altering the original and widely tested designs of USBR basins II and III (Peterka 1978, Hager 1992). These chute blocks are small baffles at the basin entrance, intended to mix the flow via jet trajectory modification in order to shorten the hydraulic jump without requiring additional tail-water (Peterka 1978, Vischer and Hager 1998). The effects of chute blocks, including a shortened and more stable hydraulic jump, were recently numerically analyzed in Valero et al. (2015) for both design and adverse conditions (i.e., insufficient tail-water).

In addition to shortening the hydraulic jump, basin accessories exert a stabilizing effect and, when correctly designed, can greatly improve stilling basin performance and economics. Omitting the chute blocks in the design could alter the performance of the baffle block in the USBR type III basin and reduce jump stability, yet omission may be compensated by the spillway steps. A USBR type III basin, as modeled in the present work, is sketched in Figure 1. Depending on project objectives, anticipated risks, and economics, stilling basins are sometimes designed to contain the entire length of the hydraulic jump for the selected design discharge of the stilling basin; consequently, any change in the flow conditions from this design point can be critical for satisfactory stilling basin performance for a range of anticipated flows. The recent study by Reclamation (Frizell and Svoboda 2012) focused on this issue by analyzing conjugate depths ratios for a USBR type III basin with a stepped spillway at the inlet. Likewise, pressure heads for similar configurations have also been presented (e.g., Meireles et al. 2010, Bung et al. 2012). Frizell et al. (2009) suggested that additional research was needed to optimize the design of stilling basins. Indeed, inlet flow properties caused by stepped chutes can vary, and little is known about the new hydraulic jump flow structure or the quantitative effect of each basin element. Use of Computational Fluid Dynamics (CFD) tools may help to gain insight into the effect that a stepped chute may have on stilling basins, including basin geometry and tail-water (Shearin-Feimester et al. 2015), given the intrinsic limitations of experimental measurements. However, lack of validation and verification is still an issue noted by several researchers (Chanson and Lubin, 2010; Chanson, 2013). Therefore, physical models of stepped chutes with stilling basins at medium and large scales (e.g.: Frizell and Svoboda 2012) may constitute an enlightening workbench for numerical models.

Different options are available for numerical models, which depend on the part of turbulence that is calculated or simulated; it is RANS modelling with eddy viscosity models that remains most commonly used for high-velocity and large domain engineering applications (Davidson 2015). More complex and time consuming methods exist; however, it is often not recommended to expend additional computational costs to obtain average forces, distributions, and velocity fields (Bradshaw et al. 1996). However, given the complexity of turbulence, no single turbulence model can reproduce all turbulent problems (Bradshaw et al. 1996, Pope 2000, Wilcox 2006, Hirsch 2007). Despite existence of models that are more complex than eddy viscosity based models (i.e., Reynolds-Stress Transport models or RST), the continued use of one and two-equation turbulence models is expected (Slotnick et al. 2014). Also, RST models lack robustness and are, on occasion, less accurate than standard RANS models (Slotnick et al. 2014).

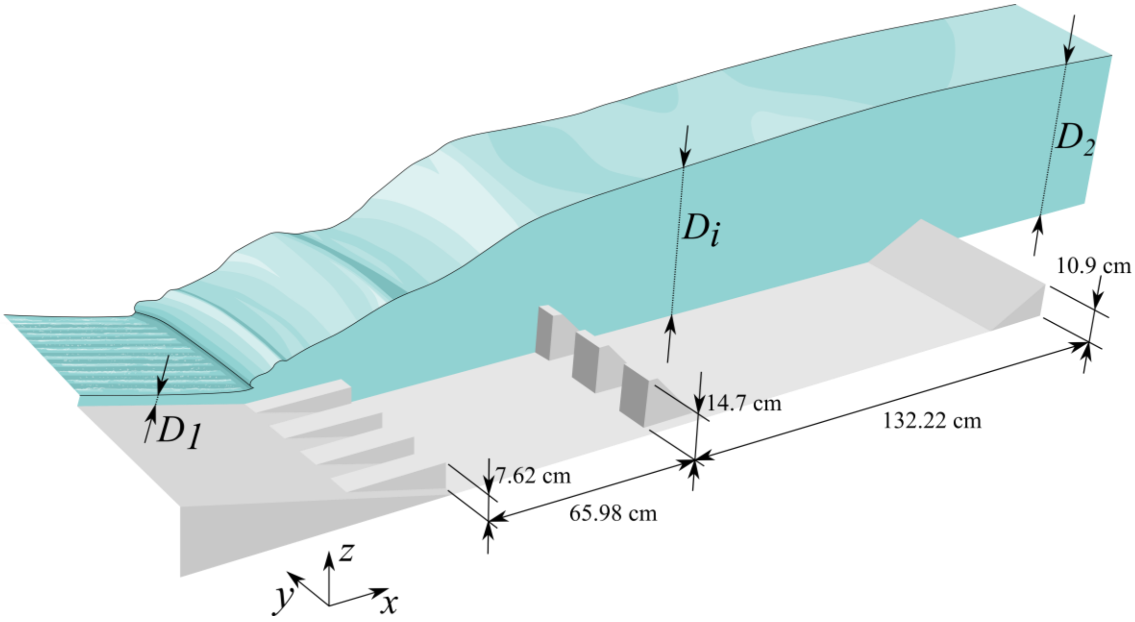

Figure 1. USBR type III basin half section and main geometrical parameters. Definition of the upstream section (S_1), intermediate section (S_i) and downstream section (S_2) with corresponding flow depths D_1, D_i and D_2.

Concerning RANS applications in related hydraulic engineering problems, Bombardelli et al. (2011) studied the accuracy of k-ε closure for RANS modelling of steep, stepped spillways using a 2D VOF single fluid approach, obtaining a good approximation of the inception point location and the non-aerated flow velocity profiles. Likewise, Valero and Bung (2015) studied the accuracy of RNG k-ε with similar conclusions for the non-aerated region with a full 3D approach. However, they noticed some difficulties for the aerated region even when using a calibrated subscale model for the air entrainment modelling (Valero and García-Bartual 2016). Wang et al. (2015) studied jet regimes downstream of a spillway flow by means of Spalart-Allmaras (SA)-DES (Detached Eddy Simulation with one transport equation for eddy viscosity) and compared their results to a 1:25 physical model. They highlighted the importance of numerical inlet conditions over the whole flow behaviour. Carvalho et al. (2008), based on a 2D RANS model and RNG k-ε for turbulence modelling, stated that RANS is appropriate for hydraulic jump problems despite its limitations (i.e., observed differences in velocities and free surface profile). Later, Valero et al. (2015) found good qualitative agreement with the study of Peterka (1978) for USBR type I and type II basins, and the study of Kawagoshi and Hager (1990) for the B-jump case , and the study of Ohtsu and Yasuda (1991) using a 3D RANS model with RNG k-ε closure. Recently, Witt et al (2015) studied accuracy of realizable k-ε for three Froude number hydraulic jumps simulations, noticing important mesh sensitivity.

In this study, eight different numerical models were developed to study the effects of a smooth chute and a stepped chute on the USBR type III stilling basin. The RNG k-ε model, which can present some advantages over other $k - \varepsilon$ models (Bradshaw, 1997), and the VOF method as described by Hirt and Nichols (1981) - used extensively for hydraulic engineering problems (Carvalho et al. 2008, Bombardelli et al. 2011, Oertel and Bung 2012) - have been employed using FLOW-3D® to properly reproduce turbulence and free surface effect upon the mean flow. First, a thorough mesh sensitivity study has been performed. Model geometry (as shown in Figure 1) and flow boundary conditions are based on the previous experimental study of Frizell and Svoboda (2012). The aim of the present work is to provide the following:

- Description and analysis of USBR type III basin performance downstream of stepped spillways under design and adverse conditions.
- Comparison with smooth spillway classical case, focused on stilling basin performance in response to a stepped chute inlet and the influence of each basin element on the hydraulic jump.

2. NUMERICAL MODEL

2.1. General Settings

The stilling basin geometry corresponds to the experimental study of Frizell and Svoboda (2012), designed for Froude number F = 8 with incoming supercritical flow clear water depth D_1 of 0.0762 m. Two geometric cases are investigated: a smooth chute and a stepped chute located upstream of the basin. Also, two inlet chute slopes have been considered: 4H:1V and 0.8H:1V. For the stepped chute, a constant step height h = 38.1 mm has been considered. In order to better observe 3D effects, the chute width has been expanded to 1.056 m (+60 %) instead of the original width used in the study of Frizell and Svoboda (2012). Main basin dimensions are shown in Figure 1. The range of simulated Froude numbers (3.1 to 9.5) is similar to the range of specific flow discharges and flow depths of Frizell and Svoboda (2012). Further details of the simulated cases can be found in Table 1.

Downstream water level (D_2) is set to the value provided by the Bélanger equation (${D_2}^*$), which is based on the upstream water level and Froude number (Chanson 2009) in order to be similar to design conditions, and it is held for 2 seconds after asteady state condition is achieved. Then, D_2 is decreased exponentially, up to 40% of its initial value during 30 additional seconds, forcing the hydraulic jump to sweep out at some intermediate instant, allowing for an assessment of the corresponding D_2 level, which causes sweep out of the hydraulic jump for adverse conditions.

Table 1. Description of conducted simulations, each ID corresponds to two simulations: the design (steady) conditions and also the adverse (transient) conditions.

ID	Inlet chute	F	D_1 [m]	Slope [H:V]
S01	smooth	3.72	0.082	4:1
S02		6.37	0.065	4:1
S03		6.17	0.064	0.8:1
S04		9.52	0.066	0.8:1
R01	stepped	3.12	0.103	4:1
R02		4.20	0.093	4:1
R03		6.47	0.080	0.8:1
R04		8.27	0.072	0.8:1

For all simulations, the RNG k-ε turbulence model has been employed (Yakhot and Orszag 1986, Yakhot et al. 1992) with the VOF method, as defined by Hirt and Nichols (1981), for free surface tracking. In addition, a single fluid approach for free surface flows was selected, thus not explicitly considering the effect of air on the water surface (Prosperetti and Tryggvason 2007). Additionally, an air entrainment routine was not considered; thus self-aeration via bubble entrainment was not considered; however, the numerical models do include the effects of any entrapped air due to free surface roughness. Solids are represented as obstacles by means of the FAVOR porosity-based technique (Hirt and Sicilian 1985). Advection is computed with a second order scheme with a slope limiter that ensures proper gradient conservation (Van Leer 1977, Hirsch 2007), which was found to be very sensitive to initial flow conditions in the stepped chutes cases. Numerical stability was ensured by setting CFL = 0.75 so that no flow property can be advected further than 0.75 cells (Hirsch 2007). Linear systems are numerically solved by using the projection based algorithm GMRES (Saad 2003) with a Krylov subspace dimension of 15. This value affects computational time but not accuracy and is chosen based on the authors' experience.

2.2. Meshing and Boundary Conditions

Flow rate has been imposed in the upstream mesh plane (inlet boundary) with a thin plate that simulates the effect of a gate (to approximate the jet box utilized by Frizell and Svoboda 2012). This plate is located a sufficient distance upstream of the stilling basin to guarantee that a fully developed flow is established upstream of the stilling basin (i.e., upstream of the chute blocks for the smooth chute case). In the downstream mesh plane (outlet boundary), a pressure condition is imposed that allows the establishing of a flow depth value with a hydrostatic pressure distribution. Lateral planes are considered as smooth walls, similar to the physical model geometry surface of Frizell and Svoboda (2012).

For the mesh sensitivity analysis, two meshing strategies have been studied: first, uniform regular cubic cells (Δx=Δy=Δz) and second, mesh refinement over the basin blocks with three linked meshes. In the second approach, one mesh block was defined for the inlet region, a second mesh block was defined for the outlet region and downstream boundary condition, and the intermediate block comprised the main region of interest with finer cells near to the basin floor. Cell counts increased structurally following the suggestion of Celik et al. (2008) regarding mesh refinement. As shown in Figure 2, the second meshing strategy considerably reduced uncertainty of the numerical solution (implicitly shown in the scattering and trend of the results). This procedure has been carried out both for case S01 and S02 obtaining similar results; for clarity reasons, only case S01 is presented herein. Results from the mesh sensitivity analysis are shown in Figure 2. Detailed descriptions of employed meshes can be found in Table 2. For the remaining cases, lengths of the meshes have been modified to cover the entire flow region and the number of cells have been adjusted to remain with same minimum, maximum, and mean cell sizes.

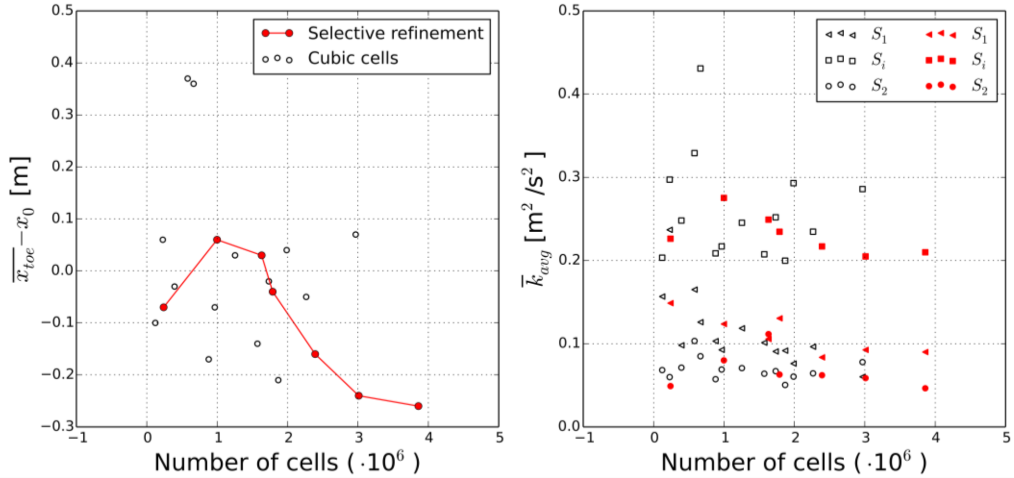

Figure 2. Time averaged hydraulic jump toe location $\overline{x_{toe}}$ relative to the inlet-basin junction location x_o and time and depth averaged turbulent kinetic energy $\bar{k}_{avg}$. White markers correspond to regular cubic cells mesh whereas red markers correspond to the multiblock refined meshing strategy for case S01.

Table 2. Main characteristics of the employed meshes for case S01, resulting from the Figure 2 sensitivity analysis.

	x		y	z		
	Length [m]	Δx [m]	Δy [m]	min(Δz) [m]	max(Δz) [m]	mean(Δz) [m]
Mesh 1	1.5	0.012	0.012	0.012	0.012	0.012
Mesh 2	3.5	0.012	0.012	0.003	0.020	0.011
Mesh 3	1.2	0.015	0.015	0.006	0.025	0.015

3. RESULTS AND DISCUSSION

3.1. General Remarks

For each case, a simulation has been conducted for t = 15 seconds in order to obtain the steady solution. Later, a new simulation has been set using the original simulation as the initial conditions. The initial two seconds of the new simulation are run with a constant downstream level $D_2 = D_2{}^*$, similar to design conditions. The solution is averaged with a temporal window $\Delta t = 0.05$ s, which allows for proper accounting of the naturally occurring oscillations effect of Unsteady RANS simulations (Spalart 2000). After those initial two seconds, the tail-water level is smoothly reduced for another t=30 seconds, procuring different solutions for adverse hydraulic conditions ($D_2 < D_2{}^*$).

Integration over a plane perpendicular to the flow was performed to obtain the real Froude number and the section representative flow depth. This becomes more important for the stepped spillway cases (R01-R04), which display greater free surface roughness.

3.2. Flow Structure

Supercritical flow enters the stilling basin and forms a roller and a shear region, similar to a classic hydraulic jump (Chanson and Brattberg 2000), as shown in Figure 3. However, it has sometimes been reported that hydraulic jumps over sloping aprons can, under certain conditions, behave more similarly to the classic wall jet. Nevertheless, some differences might be pointed out.

When the high-velocity flow impinges the roller and a smooth chute is considered, the chute blocks immediately downstream affect the flow by increasing turbulence, which can surpass the turbulence generated in the hydraulic jump impingement point. When a stepped chute is considered, the flow is continuously fed with turbulence throughout the spillway and the turbulence generated in the impingement is smaller than in the previously mentioned case.

For both types of simulated aprons, the majority of turbulent kinetic energy is held between the basin inlet and the baffle blocks or baffle piers. Additionally, the roller and its spatial location are influenced by flow impacting the baffle blocks. The end sill has proven to be effective in protecting the adjacent channel bed via flow uplift, as in the USBR type II basin numerical study of Valero et al. (2015). Note that the geometry of the USBR type III end sill is altogether different (spatially uniform) than the type II, which mimics the shape of the baffle blocks with vertical faces and angled dentations.

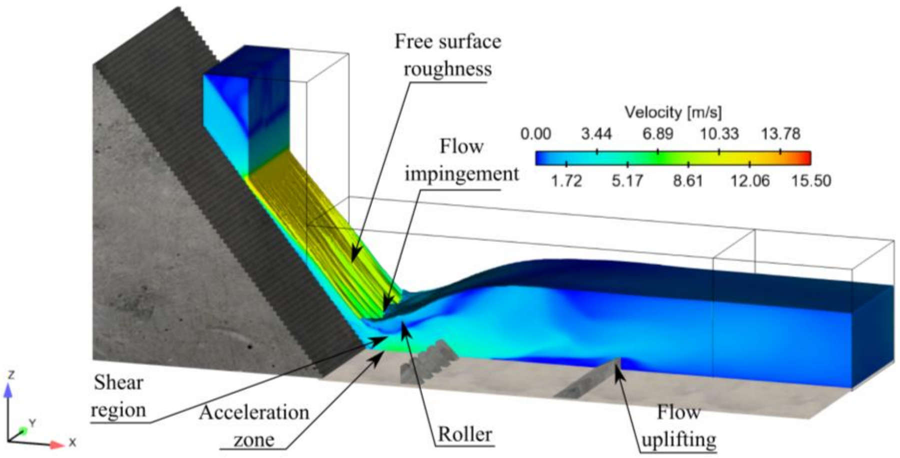

Figure 3. Instantaneous flow structure for case R03. Downstream water level set to D_2^*. Meshes are shown with black solid lines

As noted in published literature, chute blocks provide a minor jet deflection and create small but intense shear regions. Baffle blocks are more influential as an obstruction, with fluid impact magnifying 3D flow patterns and turbulence.

3.3. Velocity Decay

High velocities in the chute are rapidly reduced when flow enters the basin. Given the diffusive nature of the flow equations, it is more likely that the maximum velocity within the profile would be more resistant to velocity reduction; thus, it has been previously studied in many works as a descriptor of the flow structure.

Figure 4 shows the decay of the maximum velocity obtained at the numerical model for the studied configurations after a previous averaging in time of the velocity field. Herein, $\bar{v}_{max}$ is the maximum velocity of the averaged velocity distribution in the inlet section and, for ease of notation, $\bar{v}$ directly corresponds to the maximum velocity located at a transversal section placed in x. For context, other similar flows have been added to Figure 3 of type I and type II USBR basins from previous works with F noted. Empty markers correspond to the smooth chute at the basin inlet and filled markers to stepped chute. Although differences in velocity decay can be caused by differences in the numerical model settings, it is clearly acknowledged that a significant gap exists (as expected) between the USBR type I or II basin and the case considered herein. Baffle blocks appear to provide an improvement in the jet velocity dissipation for the USBR type III basin.

The numerical simulations identified the acceleration zone after the jet impacts the basin floor, which was also noted in the physical model study of Ohtsu and Yasuda (1991). However, when the jet impacts the baffle blocks, a more appreciable abrupt "step" is produced in the velocity decay. Within this short zone, the maximum velocity is held between the blocks and the decaying velocity trend is located immediately downstream of the baffle blocks. Considering the stepped chute, there is a noticeable effect from the steps that caused an even higher decay of the maximum velocity, possibly driven by the fact that the inlet flow is considerably more turbulent with a greater rate of energy dissipation. This is more pronounced for lower Froude numbers and can have an appreciable influence on basin performance under adverse conditions, as discussed in the following section.

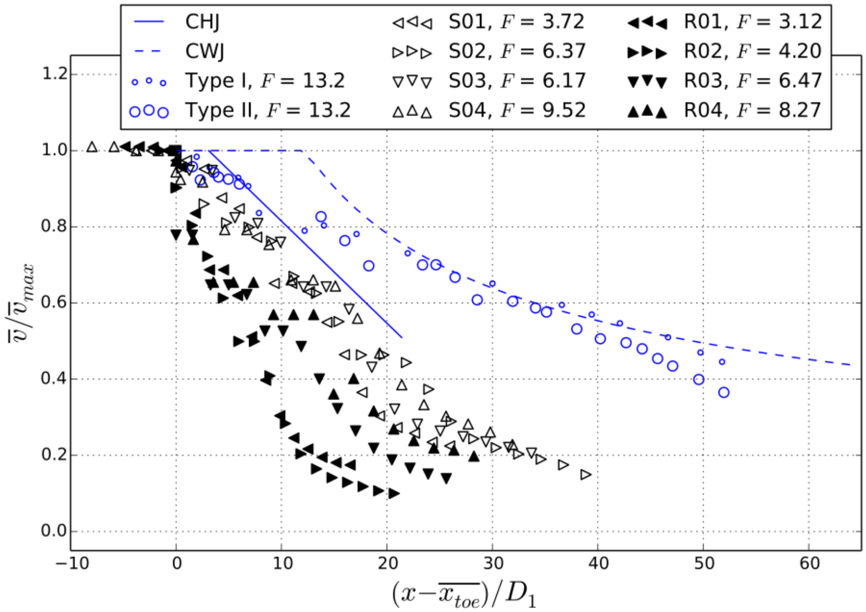

Figure 4. Decay of the maximum mean velocity for simulated cases, classic hydraulic jump (CHJ) according to Chanson and Brattberg (2000), classic wall jet (CWJ) according to Rajaratnam (1976) and the numerical simulations of USBR type I and II basins of Valero et al. (2015).

3.4. Stilling Basin Performance Under Adverse Conditions

The flow structure changes in response to a reduction in tail-water ($D_2 < {D_2}^*$), and as previously mentioned, the flow structure is also influenced by basin elements. Herein, an analysis is conducted of what are considered the most influential basin elements.

When applying Newton's second law to a control volume around the hydraulic jump, the following equation for horizontal forces balance can be obtained:

$$\iint p_1\, dA_{1,x} + \iint \rho\, {v_1}^2\, dA_{1,x} = \iint p_2\, dA_{2,x} + \iint \rho\, {v_2}^2\, dA_{2,x} + F_x \tag{1}$$

where ρ is the fluid density, $dA_{1,x}$ and $dA_{2,x}$ are the horizontal component of the area differential at sections S_1 and S_2, respectively (see Figure 1), p is the pressure at each section, and F_x is the horizontal contribution of all the other forces; for instance, the resulting baffle blocks force (F_b), the force opposed by the end sill (F_{es}) or the force exerted by the basin bed (F_w), among others. For a control volume over a slope, gravity may also contribute. F_x could also be considered the force caused by the chute blocks or the lateral walls; however, they have not been considered in this study.

P_1 and P_2 are used to represent the resulting horizontal pressure forces. For the momentum contribution, F_{m1} and F_{m2} are employed. Then, the inlet flow contribution can be written as P_1 + F_{m1} and, when no other force is considered, P_2 + F_{m2} may be an equivalent sum when a classical hydraulic jump is considered.

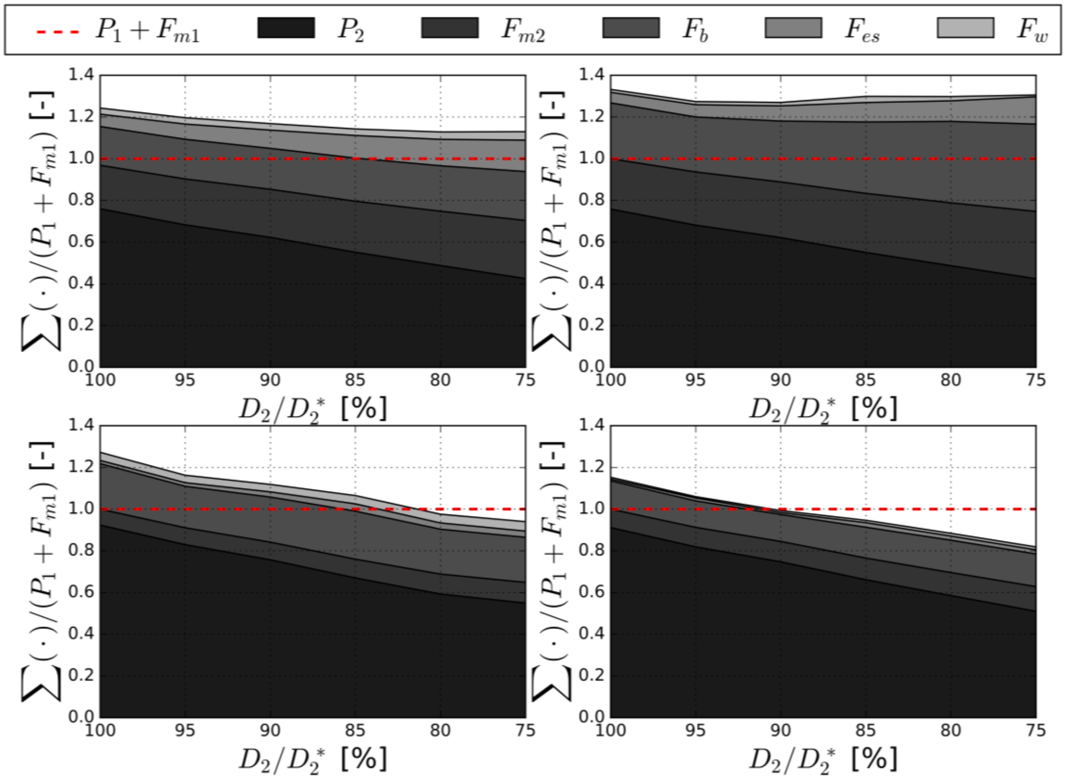

Figure 5. Relative contribution of each basin element to the stability of the hydraulic jump for different adverse tail-water levels, up to 75% of design downstream depth D_2^*. Case S01 (upper left), R01 (upper right), S04 (lower left), R04 (lower right). Red dashed line represents inlet flow contribution.

Figure 5 shows the comparative importance (relative to the inlet flow characteristics) of each element for the lower and higher studied Froude numbers. The effect of each term is presented in a cumulative way for each D_2/D_2^* considered. Please note the significant importance of the baffle blocks term in all cases, which can contribute more than the downstream momentum. When tail-water is decreased, flow over the baffle block provides an increased contribution to the total force, especially for the lower Froude number cases. This phenomenon is of special interest when combined with the stepped spillway since the baffle block term helps to overcome the contribution of the inlet flow. This may be caused by the important velocity decay observed for this configuration in Figure 4. As a consequence, total downstream forces remain higher than P_1 + F_{m1} for high reductions of tail-water level; this might explain the highly stable hydraulic jump observed by Frizell and Svoboda (2012) for Froude numbers below 6. The effect of basin bed friction is slightly smaller for the stepped spillway cases, which may be due to the steps elevating the centroid of the velocity profiles (and resulting in lower gradients close to the channel bed).

4. CONCLUSIONS

CFD may be utilized to analyze the complex behaviors of flows within a hydraulic jump stilling basin, providing qualitative and quantitative information to practitioners, in part through comparative analyses. Unsteady RANS equations have been employed with VOF and RNG *k*-*ε* for free surface tracking and turbulence modeling, respectively.

Eight different Froude numbers (F) ranging from 3.1 to 9.5 have been analyzed for a stilling basin designed for F = 8, following recent studies conducted by Reclamation. The basin flow structure was investigated for both smooth chute and stepped chute cases, each at two slopes. Additionally, the modelled basin has been tested for design and adverse hydraulic conditions, obtaining a detailed insight of the role of each basin element and their adapting roles when insufficient tail-water conditions exist.

When a stepped chute is considered, the flow is continuously fed with turbulence throughout the spillway chute, and the turbulence generated via impingement is smaller than for a smooth chute. The steps cause an even higher decay of the maximum velocity within the basin, possibly driven by the fact that the inlet flow is considerably more turbulent relative to a smooth chute. This decrease is more pronounced for smaller Froude numbers. Also, baffle blocks promote maximum velocity decay. The acceleration zone immediately downstream of the basin entrance is more significant in the profile when the jet impacts baffle blocks.

In Figure 5, the relative importance (relative to the inlet flow characteristics) is shown for each stilling basin element for the lower and higher studied Froude numbers. The effect of each term is presented in a cumulative way for each D_2/D_2^* considered. The influence of baffle blocks was found to exceed that of the other stilling basin elements.

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