International Junior Researcher and Engineer Workshop on Hydraulic Structures, 2006, J. Matos and H. Chanson (Eds), Report CH61/06, Div. of Civil Eng., The University of Queensland, Brisbane, Australia - ISBN 1864998687

LABYRINTH SPILLWAYS: COMPARISON OF TWO POPULAR U.S.A. DESIGN METHODS AND CONSIDERATION OF NON-STANDARD APPROACH CONDITIONS AND GEOMETRIES

Greg Paxson
Schnabel Engineering, Inc., U.S.A., gpaxson@schnabel-eng.com
Bruce Savage
Assistant Professor, Idaho State University, U.S.A., savabruc@isu.edu

Keywords: Labyrinth Spillways, Computational Fluid Dynamics (CFD), Dams

Abstract : Extensive physical modelling of labyrinth spillways, primarily flume studies, has been performed, resulting in the development of several design methods. Two of the more common methods used in the U.S.A., referred to herein as the Lux and Tullis methods, are compared for a given labyrinth geometry. Results of a RANS Computational Fluid Dynamics (CFD) model, using commercially available software (Flow-3D), for the same configuration is shown to give results comparable to those obtained using these design methods. Non-standard approach conditions and geometries are modelled using physical and numerical methods and the applicability of the Lux and Tullis methods for these conditions is evaluated. In particular, this paper considers the effects of aspect ratios (W/P) less than the minimum values recommended for the Lux and Tullis design methods. The results indicate that while the design methods are not appropriate for aspect ratios less than 2; hydraulic performance does increase for lower W/P values. Further testing to define labyrinth hydraulic behaviour for these lower W/P values is recommended.

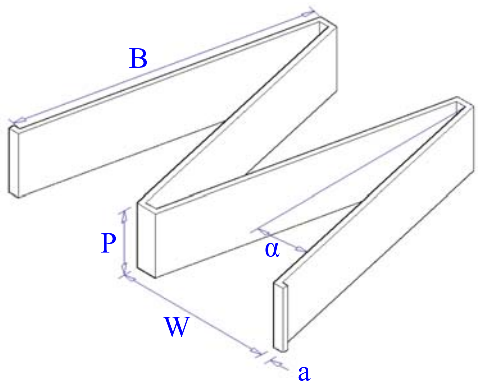
INTRODUCTION

In the U.S.A., both spillway design flood (SDF) requirements and meteorological estimates of floods have increased for existing and new dams. Labyrinth spillways can be an economical solution for increasing spillway capacity as they provide increased unit discharge over conventional weirs for a given head (Darvas, 1971). The authors have observed an increase in the use of labyrinth spillways in the U.S.A. over the last 25 years. It is likely that this increase in partly due to the development of design methods by Lux and Hinchliff (1985) and Tullis *et al*. (1995), referred to herein as the Lux and Tullis methods. Prior to the development of these methods, Taylor (1968) extensively studied the behaviour of labyrinth weirs and presented the hydraulic behaviour as it compares to that of a sharp-crested weir. Hay and Taylor (1970) followed up on Taylor's work and developed design criteria for labyrinth weirs. Additional work by Darvas (1971) utilized the results from physical model studies to expand on the theory and developed a family of curves to evaluate spillway performance. Falvey (2003) summarizes the Lux and Tullis methods, which were empirically developed from physical model studies (readers should note that the Lux equation is incorrectly printed in the Falvey (2003) publication and should refer to Lux and Hinchliff (1985) for the correct equation).

Figure 1 shows the parameters commonly used define a trapezoidal labyrinth weir. The developed length, L, of a weir is generally defined as the total length along the crest.

The hydraulic performance of labyrinth weirs is partially dependent on the parameters shown in Figure 1. Other factors affecting performance include crest shape (sharp crested, quarter or half round, etc.), approach conditions of the weir (located in a channel or projecting into reservoir, etc.), and downstream conditions (i.e. slope and geometry of the downstream channel).

Fig. 1 – Two-cycle labyrinth weir and parameters



W: Cycle Width
P: Weir Height
B: Sidewall Length
α: Sidewall Angle
a: Half Apex Width
L: Length of Weir (single cycle)
H: Total Upstream Head on Weir
n: Number of Cycles

APPLICATION OF LUX AND TULLIS METHODS

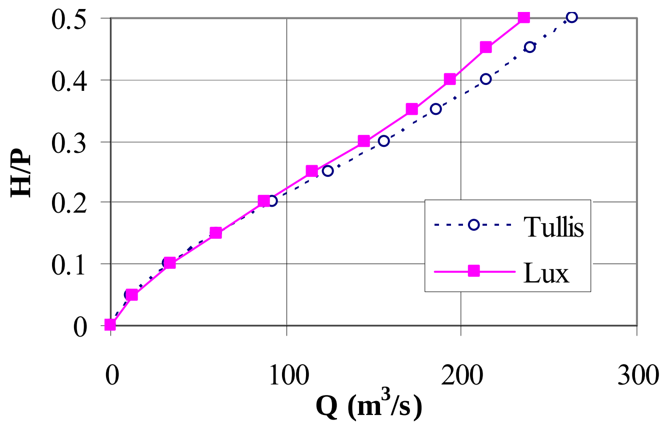
The Lux and Tullis methods were empirically developed for design of labyrinth spillways using a wide range of weir geometries. For both methods, a quarter-round crest shape is recommended. Tullis recommends a W/P ratio of 3 to 4 while the Lux method is considered appropriate for W/P ratios greater than 2. Both methods suggest a maximum H/P of about 0.7; however, the equations of Tullis are considered applicable up to H/P of 0.9.

To compare these two methods, discharge rating curves were developed for a given labyrinth weir. The Hyrum Dam auxiliary labyrinth spillway geometry was selected for this comparison (see Table 1 for dimensions). Hyrum Dam is located in Utah, U.S.A. and a labyrinth spillway was proposed to increase the discharge capacity of the structure. Extensive physical model studies were performed by Houston (1983) to evaluate various labyrinth geometries and approach conditions. The spreadsheet presented on page 90 of Falvey (2003) was used for the Tullis method while a spreadsheet for the Lux method was developed by the authors. The computed discharge rating curves for the Lux and Tullis methods are plotted in Figure 2.

Table 1 – Hyrum labyrinth geometry

Weir Height (P)	3.66 m	Apex Width (2a)	1.22 m
Cycle Width (W)	9.14 m	Number of Cycles (n)	2
Aspect (W/P) ratio	2.5	Length Magnification (L/W)	4.95

Fig. 2 – Computed rating curves



The methods provide very similar results for H/P ratios between about 0.1 and 0.2, where the flow is considered "fully aerated" and the labyrinth acts similar to a linear weir (Lux and Hinchliff, 1985). For the design head of the Hyrum Labyrinth (1.83 m or H/P of 0.5), the Lux estimate of discharge is about ten percent less than that of the Tullis method. This difference is not considered significant given the empirical nature of the methods; however, further comparisons of the methods are considered warranted.

COMPUTATIONAL FLUID DYNAMICS

In recent years, advances in computational power have brought out of obscurity a numerical modelling branch in fluid mechanics referred to as Computational Fluid Dynamics (CFD). In general, CFD numerically solves the Navier-Stokes equations for fluid flow using a variety of methods. Advances in this area provide another tool for engineers to evaluate spillways. As computing power continues to increase and algorithms improve, CFD may offer the ability to evaluate an individual spillway at a cost less than a physical model. In addition, CFD allows for easy extraction of additional information across the computational domain such as forces, velocities, and pressures. Collection of this data in a physical model can be cost and time prohibitive. One of the main disadvantages of this new tool is that its accuracy is largely unproven, although studies show the capability is promising. Ho *et al*. (2001, 2003) compared their numerical results for the USACE standard ogee crest with and without piers and reported results similar to physical model studies. Savage and Johnson (2001) and Savage *et al* (2001) validated CFD results against different physical model studies and found comparable results for the computation of the discharge and crest pressures. Other studies such as Yang *et al*. (1998) and Gessler (2005) show an increased use of CFD in modelling spillway performance.

One of difficulties of numerically modelling flow over spillways is tracking the free surface especially with rapidly varied flow such as the flow changing from subcritical to supercritical. Flow over a weir can complicate this problem when the nappe springs away from the weir, creating a jet with an air/water interface on the top and bottom of the jet that needs to be tracked.

To analyze this problem, the commercially available CFD code, Flow-3D by Flow Science was selected. Flow3-D is known for its ability to accurately tracking the free surface using the Volume-of-Fluid (VOF) method. The original VOF method is outlined by Hirt and Nichols (1981). The VOF method tracks the free surface by defining computational cells empty, partially full, or full of fluid. The free surface of the fluid in a cell is defined by a plane and its orientation is defined by the surrounding cells and their volume of fluid. A similar method called Fractional-Area-to-Volume-Ratio (FAVOR) is used to define the labyrinth within the model. The FAVOR method is outlined in Hirt and Sicilian (1985). The computational domain is defined by a hexagonal region that is then subgridded or discretized into cells. The VOF and FAVOR modified Reynolds-averaged Navier-Stokes (RANS) equations are solved using a finite volume method. The Renormalized Group Theory (RNG) model was used for turbulent closure (Yakhot and Orszag, 1986).

To compare the Lux and Tullis methods with the Flow-3D model, a 3-D solids model having the geometric parameters shown in Table 1 was constructed and imported into the program. The model was placed into a gridded domain simulating a labyrinth placed within a channel. This layout is considered similar to that of the experiments used in the development of the Lux and Tullis methods. The upstream boundary in the numerical prototype was located approximately 90 m from the weir. This allowed sufficient length for a flow profile to be established in the channel. In addition, to resolve the flow field more accurately in the area closest to the labyrinth weir, smaller cell sizes were used in regions of rapidly varied flow by using nested blocks. A nested block is a defined hexagonal subregion within the computational domain. Smaller cell sizes result in an increase in the number of cells but the additional cells refine the flow field, more accurately determining pressures, velocities, and the location of the free surface. As the computations proceed, information is passed between the outer block and the inter blocks.

The numerical results are plotted in Figure 3 along with those obtained using the Lux and Tullis methods. The results indicate Flow-3D provides results that match closely with the design methods.

Fig. 3 - Comparison of numerical results for a labyrinth in a channel with Lux and Tullis methods

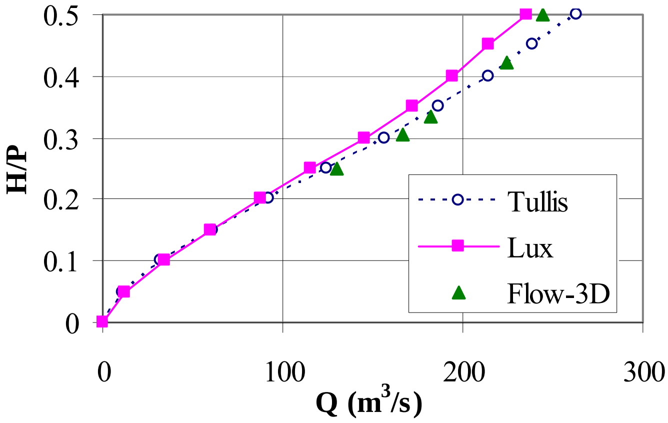
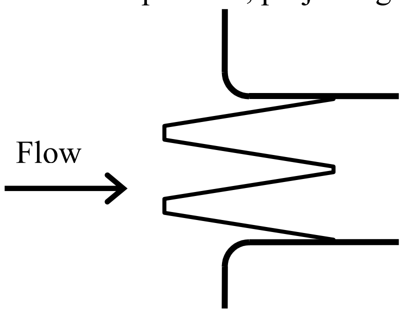


Fig. 4 – Plan of Hyrum final design: inverted position, projecting into reservoir, curved entrance



APPROACH CONDITIONS

A design limitation of the Lux and Tullis methods is that approach effects on the discharge are not considered, as both methods were developed using data from flume studies. Because labyrinth spillways are not always placed in a channel; approach conditions can be significantly different than what would be produced in a flume. In the physical model studies for the Hyrum spillway, Houston (1983) evaluated several different labyrinth configurations and approach conditions that were not considered traditional "flume" configurations. Configurations included locating the weir flush with the entrance from the reservoir, 1.5 m downstream of the entrance, and projecting into the reservoir. Labyrinths in both the "normal" and "inverted" positions (Falvey, 2003) with various entrance conditions (square, curved, embankments at entrance) were modelled. The most efficient configuration studied was a labyrinth projecting into the reservoir with a curved channel entrance, as shown in Figure 4. This will be referred to as the Hyrum final design.

Incorporating non-standard approach conditions into a design increases the complexity of the hydraulic behaviour and it is likely that the labyrinth performance cannot be accurately modelled using the Lux or Tullis design methods. Physical modelling of such a design would be more costly than a flume study because it would require construction of a large head box to model the approach geometry of the reservoir. For these situations, CFD modelling could prove be a cost effective alternative. Table 2 shows results for the Hyrum final design geometry using the original physical model study data, Flow-3D, and the Lux and Tullis methods.

Table 2 – Physical model, Flow-3D, and design Method Results, Hyrum final design geometry at design head of 1.83 m

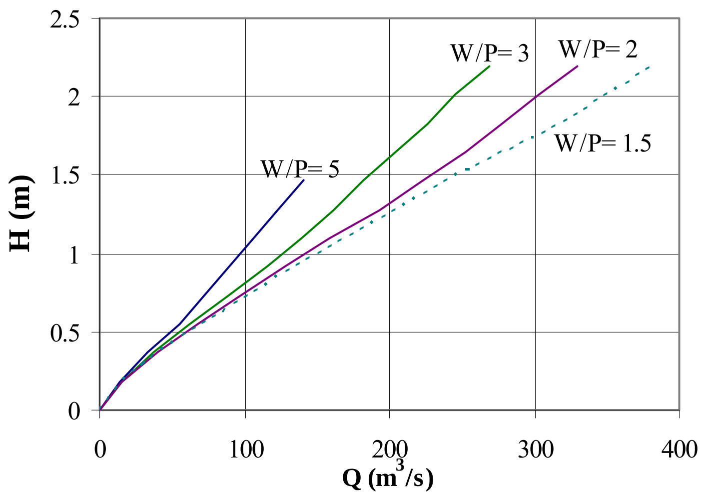
Model/Method	Discharge (m^3/s)
Physical Model (Houston, 1983)	278
Flow 3-D	264
Lux method	236
Tullis method	263

These results show agreement between the numerical and physical model within about 5% for the conditions evaluated. The application of the Tullis method yields results similar to those of the CFD model while application of the Lux method results in a lower computed discharge.

THE ASPECT RATIO

As previously discussed, the Lux and Tullis methods recommend a minimum aspect ratio, or W/P of 2 to 3. Falvey (2003) states that the studies of Hay and Taylor (1970) showed that the aspect ratio does not have a significant effect if it is greater than 2; however, the application of either the Lux or Tullis methods indicate that hydraulic performance does improve with increasing weir height (P), and corresponding decreases in the aspect ratio for a given plan geometry. To illustrate this, only the weir height (P) of a labyrinth having the Hyrum geometry was adjusted and discharge rating curves were developed using the Lux method. The results are presented in Figure 6, indicating that increasing the weir height for a given plan geometry (i.e. decreasing W/P) results in a greater discharge for a given head, especially for higher heads. It should be noted that the application of the aspect ratio of 1.5 using the Lux method is not considered appropriate since it is below the recommended value of 2. The data is presented in Figure 5 for comparison only.

Fig. 5 - Effect of the W/P ratio, Lux method

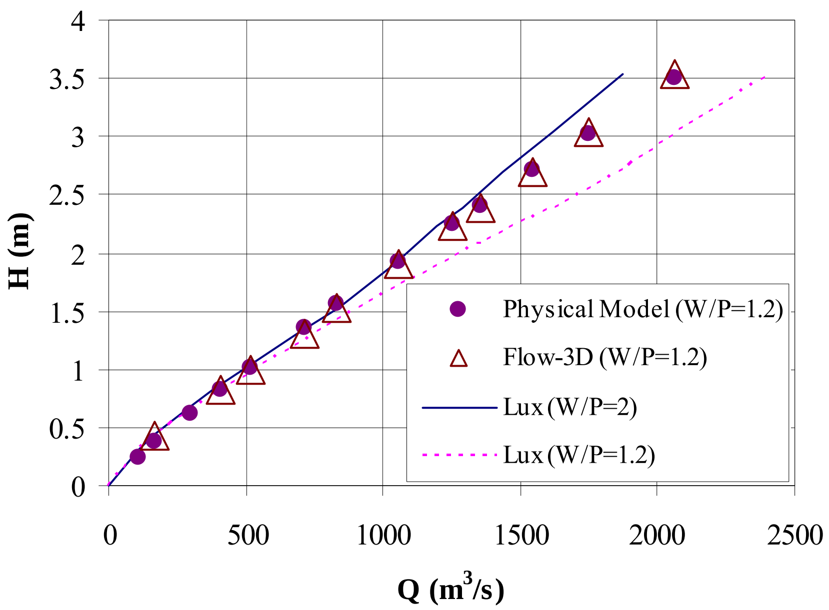


The Lux and Tullis methods could be conservatively applied to a labyrinth having an aspect ratio less than 2 to 3 by assuming a weir height less than actual to obtain the minimum recommended W/P value. For example, a labyrinth having a weir height (P) of 3 m and cycle width of 5 m (i.e. W/P=1.67) could be modelled using the Lux method with an assumed weir height of 2.5 m (i.e. W/P=2). This would likely result in lower computed discharges for the spillway but would provide for a conservative design.

Reducing the aspect ratio to less than 2 may provide benefit to a project without significantly affecting performance. The raising of an existing labyrinth weir (i.e. increase P) decreases W/P. An example of this is the Dog River Dam located in Georgia, U.S.A., where a 3 m raise of the normal pool is proposed to increase storage of this water supply reservoir (Savage *et al*. 2004). The existing labyrinth spillway will be raised from a height (P) of 4.6 m to 7.6 m.

To evaluate the effects of the higher weir (increased P, decreased W/P) both physical and numerical model studies were performed. The data from these studies is presented in Figure 6 along with estimated discharge ratings using the Lux method for the original W/P ratio of 2 and the lower W/P ratio of 1.2 corresponding to the raised weir.

Fig. 6 - Comparison of physical and numerical model studies for Dog River Dam



The results show that for this study, the Flow-3D model yields results similar to that of the physical model study. In addition, it appears that there is an increase in hydraulic performance for the lower W/P values. As expected, the application the Lux method with the "out of range" aspect ratio of 1.2 overestimates the discharge; however, the approach of assuming a smaller weir height to obtain the minimum recommended aspect ratio of 2 in the application of the Lux method underestimates discharge. The Tullis method was also evaluated and resulted in similar findings. For lower heads (H less than about 2 m and H/P less than about 0.3), the use of the Lux method with the minimum recommended aspect ratio of 2 is considered appropriate.

Fig. 7 - Full height labyrinth, Whitewater Lake Dam

For relatively small dams (<8 m high), it is often economical to construct a "full height" labyrinth, where the base slab is at the level of the embankment foundation. This construction eliminates the need for a chute to convey flow to the downstream toe and also simplifies the foundation design since the labyrinth structure is founded on natural soil as opposed to new compacted fill, which is more prone to settlement. Figure 7 is a photograph of a full height labyrinth at Whitewater Lake Dam located in Georgia, U.S.A.

For full height labyrinths, the weir height (P) is controlled by the dam height and often the maximum head (H) over the weir is small in comparison. To maintain the recommended minimum aspect ratio of 2, a large weir height (P) necessitates a wide labyrinth cycle (W), likely increasing the cost of the structure. Alternatively, the hydraulic design of the labyrinth could assume a lower weir height to maintain a W/P of 2 or more as discussed earlier. A physical model study to evaluate the hydraulic performance of the structure would likely be cost prohibitive for small dams; therefore, additional research is recommended for these lower values of W/P.

The Hyrum final design model geometry was used to further evaluate the aspect ratio using Flow-3D. The weir height was adjusted from the original model value of 3.6 m to 6.1 m, decreasing W/P from 2.5 to 1.5. Results are presented in Table 3.

The Flow-3D model produced results within about 5 percent of the physical model for the original geometry (W/P=2.5). In addition, for the conditions evaluated, the Flow-3D model results show a significant increase in discharge (14 to 19 percent) for the lower W/P value of 1.5.

Table 3 – Computed flows (m^3/s) for physical and Flow-3D models, Hyrum plan geometry

Head	**Physical Model**	**Flow-3D Model**	
(m)	**(Houston, 1983)**	**W/P=2.5**	**W/P=1.5**
1.67	259	259	295
1.83	278	264	314

CONCLUSIONS

Based on the modelling and analyses performed for the labyrinth spillway configurations presented herein, the following conclusions can be drawn:

- Two of the more common design approaches, the Lux and Tullis methods, yield similar results for the evaluation of hydraulic performance of labyrinth spillways. For the examples herein, the Lux method computes a discharge of about ten percent less than the Tullis method for a typical design head (H/P=0.5).
- The use of the Flow-3D program to model labyrinth hydraulics produced results similar to the two design methods for a weir located in a channel or flume.
- The Lux and Tullis design methods do not consider approach conditions such as a labyrinth projecting into a reservoir. For this condition, these methods typically underestimate discharge for a given head.
- Numerical modelling using Flow-3D yields results similar to the physical model for a labyrinth weir projecting into the reservoir.
- The aspect ratio (W/P) does appear to affect labyrinth performance. For the examples evaluated, both physical and numerical modelling showed increased hydraulic performance for W/P ratios less than 2; however, the design methods are not appropriate for these lower values.

The authors are continuing research using numerical methods to model non-standard labyrinth geometries and approach conditions.

ACKNOWLEDGEMENTS

The authors would like to acknowledge Frederick Lux III for providing data and insight related to his previous and ongoing work related to labyrinth spillways.

REFERENCES

Darvas, L.A. (1971) "Performance and design of labyrinth weirs." *J. Hydr. Engrg.*, ASCE, 97(8), 1246-1251.

Falvey, H. T. (2003). "Hydraulic Design of Labyrinth Weirs." *ASCE Press.*

Gessler, D. (2005). "CFD Modeling of Spillway Performance." *Proc. World Water and Environmental Resources Congress 2005*, Raymond Walton-Editor, May 15-19. Anchorage, Alaska, USA.

Hay, N., and Taylor, G (1970). "Performance and design of labyrinth weirs." *J. Hydr. Engrg.*, ASCE, 96(11). 2337-2357.

Hirt, C.W. and Nichols, B.D. (1981). "Volume of fluid (VOF) method for the dynamics of free boundaries." *J. Comp. Physics*, Vol. 39, 201-225.

Hirt, C.W. and Sicilian, J.M., (1985). "A Porosity Technique for the Definition of Obstacles in Rectangular cell meshes." *Proc. 4th International Conf. Ship Hydro.*, Nat. Academy of Science, Washington, DC.

Ho, D.K.H., Boyes, K.M., and Donohoo, S.M. (2001). "Investigation of Spillway Behavior under Increased Maximum Flood by Computational Fluid Dynamics Technique." *Proc. 14th Australasian Fluid Mechanics Conference*, Adelaide University, Adelaide, Australia. December 10-14.

Ho, D.K.H., Boyes, K., Donohoo, S., and Cooper, B. (2003). "Numerical Flow Analysis for Spillways." *Proc. 43rd ANCOLD Conference*, Hobart, Tasmania, 24-29 October.

Houston, K.L. (1983). "Hydraulic Model Study of Hyrum Dam Auxiliary Labyrinth Spillway." *Report No. GR-82-13*, US Bureau of Reclamation, Engineering and Research Center, Denver, CO, May 1983.

Lux III, F.L., and Hinchcliff, D. (1985). "Design and Construction of Labyrinth Spillways." *Proc. 15th International Congress on Large Dams*, Vol. 4, Q. 59, R. 15, pp. 249-274, International Commission on Large Dams, Paris, France.

Savage, B.M. and Johnson, M.C. (2001). "Flow over Ogee Spillway: Physical and Numerical Model Case Study." *J. of Hyd. Engrg*. ASCE, Vol. 127, No. 8, Aug. pp. 640-649.

Savage, B.M., Johnson, M.C. and Geldmacher, R. (2001). "Comparison of physical versus Numerical Modeling of Flow over Spillways." *ASDSO 2001 Annual Conference Proceedings*.

Savage, B.M., Frizell, K., & Crowder, J. (2004). "Brains versus Brawn: The Changing World of Hydraulic Model Studies." *ASDSO 2004 Annual Conference Proceedings*.

Taylor, G. (1968). "The performance of labyrinth weir." *PhD thesis*, University of Nottingham, Nottingham, England.

Tullis, J.P., Amanian, N., & Waldron, D. (1995). "Design of labyrinth spillways," *J. of Hyd. Engrg*. ASCE, Vol 121, No. 3, pp. 247-255.

Yakhot, V. and Orszag, S.A., (1986). "Renormalization group analysis of turbulence. I. Basic theory." *J. Scientific Computing*, 1(1), p 1-51.

Yang, J., Hemstrom, B., and Blain, W.R. (1998). "CFD: a useful tool in spillway capacity determination", *Proc. International Conf. on Hydraulic Eng. Software*, Lake Como, Italy. Sept. vol. 4, pp. 237-246.