Solution of Flow Field Equations to Investigate the Best Turbulent Model of Flow over a Standard Ogee Spillway in Finite Volume Method

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Abstract

Ogee spillways are the most important structures used to control floodwater, and built at the same time concrete or masonry dams are constructed. Due to such an importance, they shall be studied. The development of computer science and different types of computational fluid dynamic software, the behavior of ogee spillways can be studied in a short time and without paying high expenses. In this research, numerical evaluation and Fluent Software are used to study the impacts of different models of turbulence on the appropriate parameters of flow over ogee spillway. Based on Gauss-Seidel convergence condition, the step time of 0.01 second has been used for the convergence and control of equations. The results show that in case the RNG $k-\varepsilon$ turbulence model is used, the accuracy of the results obtained from the flow over ogee spillway is increased. The percentage of relative error of the flow discharge over ogee spillway is 9.687 indicating that this method is of high accuracy. Whereas the decision criterion (P-value) of SPSS software is about 0.000006, and is very lower than 0.05, therefore it can be claimed that this numerical study is of high precision. The comparison between numerical model and the results obtained from Flow3D software and other numerical and laboratory methods indicates that this method is of high accuracy. Therefore, the use of numerical methods is recommended.

Keywords: *Ogee Spillway, Numerical Model, Turbulent model, P-value, Validation*

1. Introduction

Due to proper function, low costs, ability to control floodwater, and high safety factor, ogee spillways are among those hydraulic structures that are used frequently. One of the most important applications of these spillways is to control the height and volume of the tank [16]. Due to their vast application, ogee spillways have been studied mostly [5], which will be pointed out in the following:

In 1965, Cassidy used numerical model in a 2D space to determine the pressure on the crest of ogee spillway based on potential flow [1]. In 1998, Olsen and Kjellesvig used Reynolds equations and standard k - ε equation in finite volume method to analyze the flow over ogee spillway in 3D and 2D spaces [2]. In 1999, Burgisser and Rutschmann used finite element method to analyze the vertical component of flow over the crest of spillway in 2D space supposing that there is incompressible and turbulent flow [3]. Tufi and Wilson used in 2001 the finite difference method to analyze vertical flow over ogee spillway crest in a 2D space assuming that there is a potential flow and Neumann condition is imposed on the boundaries of flow field [4]. Bruce et al. conducted in 2001 a comprehensive study to compare the parameters of flow over standard crested ogee spillways using a physical model, numerical model and existing studies [5]. Latif Bouhadji conducted in 2002 a numerical study on turbulent flow over spillways in a 3D space [6]. In 2003, Chen et al. modeled turbulent flow over stepped spillway using finite volume method [7]. In 2003, Ho et al. studied the maximum impacts of contingent floodwater over spillways [8]. In 2004, Jean and Mazen modeled flow over ogee spillway numerically [9]. In 2005, Dae Geun Kim and Jae Hyun Park used the commercial numerical model of computational fluid dynamics of Flow 3D software to study the properties of flow including flow rate, water surface profile, pressure imposed on the crest of ogee spillway, pressure vertical distribution and speed based on the scale of model, the impact of surface roughness and details [10]. In 2006, Bhajantri et al. studied the hydraulic model of flow over ogee spillway numerically considering downstream and the information obtained from two physical models have been compared with the results obtained from numerical study of two crested ogee spillways [12]. In 2009, Vosoughifar and Daneshkhah used numerical methods such as finite difference and finite volume to evaluate numerically the impacts and function of the surge tank of hydraulic hammer on the cover of water tunnels transferring water to concrete dam turbines [13]. In 2009, Angela Ferrari simulated numerically the flow with free surface over a spillway with sharp crest [14]. In this research, numerical anaysis of different turbulant models are compared to calculate properly the profile of flow over ogee spillway using finite volume method.

2. Research Method

In this paper, Fluent Software has been used to study the physical properties of the field of flow over ogee spillways. For this purpose, it is required to plot the geometry of model and meshing using GAMBIT software. Fluent is a multipurpose software used for the modeling of fluid flow, transfer of heat and chemical reactions. It has the capability of analyzing complicated and turbulent flows and it is based on finite volume method, which is a very strong method to solve computational fluid dynamics. This software uses volume of fluid (VOF) model to determine the free surface of flow. Nichols and Hirt provided a model to determine the common surface of two-phase fluid, which is used in many hydraulic problems [22]. In this research, 90% of the volume of cell is water and the remaining part consists of air. The equations governing flow over spillway in this software include continuity equation and Navier-Stokes equations. To solve the equations of Navier – Stokes, Reynolds averaging method (Rans) has been used in this research in such a way that the fluctuations of turbulent flow are inserted into the equations indirectly [15]. To solve the field of turbulent flow based on continuity equations and Reynolds Averaged Navier – Stokes equation, it is required Reynolds Stress Equations to be modeled using specified methods. Turbulent models have been classified based on the application of their design and number of differential equations to create relation between turbulence stresses and averaged rates or their gradients. These models include zero equation models, one equation model (Spalart – allmaras model), two equation models, algebraic stress model, Reynolds stress model, Reynolds stress models (five equation model). Among these models, two-equation model has been studied due to the satisfactory results and simplicity of application for ogee spillways [5]. This model has also been divided into the following classes: standard k – ω model (shear stress transport (sst) k – ω model) and k – ε model (standard k – ε model , realizable k – ε model , renormalization group (RNG) k – ε model) [20]. These models are able to solve two differential equations. Equation of ε or ω has been added to the equation of k. The equation of turbulence kinetic energy expresses the scale of velocity and the equation of turbulence kinetic damping rate, ε or ω, has also the scale of length.

k – ω model

k equation:

$$\frac{\partial k}{\partial t}+U_j\frac{\partial k}{\partial x_j}=\frac{1}{\rho}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta^* k\omega+\frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma^*\mu_T\right)\frac{\partial k}{\partial x_j}\right] \tag{1}$$

ω equation:

$$\frac{\partial \omega}{\partial t}+U_j\frac{\partial \omega}{\partial x_j}=\frac{\alpha\omega}{\rho k}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta k\omega^2+\frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma\mu_T\right)\frac{\partial \omega}{\partial x_j}\right] \tag{2}$$

k – ε model

k equation:

$$\frac{\partial k}{\partial t}+u_i\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{v_i}{\delta_k}\frac{\partial k}{\partial x_i}\right)+v_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j}-\varepsilon \tag{3}$$

ε equation:

$$\frac{\partial \varepsilon}{\partial t}+u_i\frac{\partial \varepsilon}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{v_i}{\delta_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right)+c_{\varepsilon1}\frac{\varepsilon}{k}p-c_{\varepsilon2}\frac{\varepsilon^2}{k} \tag{4}$$

Lander and Spalding provided the abovementioned constants based on experimental information, in form of table 1.

c_μ	$c_{\varepsilon1}$	$c_{\varepsilon2}$	δ_k	δ_ε
0.09	1.44	1.92	1	1.3

Table 1. Lander and Spalding Factors for k – ε equations

These equations were used successfully for many 2D boundary laminar flows over wall, flow inside channels, free shear flows, eddy flows, 3D boundary laminar flows, etc. [20].

2.1. Statistical Analysis Method with SPSS

The Statistical analysis method (p-value) with SPSS software has been used to compare the parameters of flow over ogee spillway. Statistical analysis method is used to confirm null hypothesis (meaningful difference) or reject null hypothesis (considerable difference). The correctness of null hypothesis (H_0) has been always emphasized, unless something contrary is proved. Significance level (α) is the error level designated by the researcher as a criteria to reject null hypothesis (H_0) (usually 5%). P – Value, which is called decision criterion, is level of error for rejecting null hypothesis and it is shown in form of *Sig (2-tailed)*. If the P-value is lesser that Alpha, null hypothesis can be rejected more easily. In this research, the lesser amount of this value proves the reality of numerical solution [17].

2.2. Two Dimensional Discretization of Equation

For the convergence of numerical modeling, it was required to control the equations and their related relations in order to define temporal steps for the software. For this purpose, the temporal steps shall be applied based on the meshing measure to achieve sustainability. The discrete differential equation of nonpermanent fluid flow is as follows:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + S \tag{5}$$

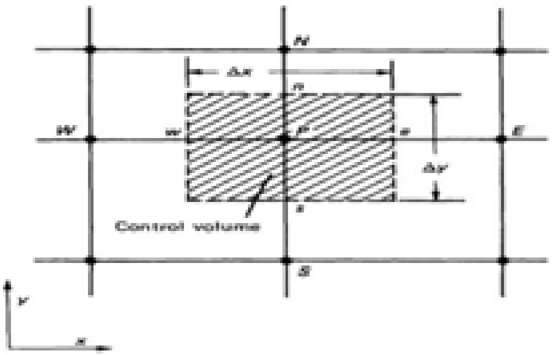

Fig. 1. Control Volume for 2D Situation

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + a_T T_T + a_B T_B + b \tag{6}$$

There are different iteration methods for solving algebraic equations, among which the Gauss-Seidel Point–by–Point Method was used. In this method, the variable values of each node have been calculated based on specified instruction. Discrete equation is as follows:

$$a_T T_P = \Sigma a_{nb} T_{nb} + b \tag{7}$$

Where, nb indicates neighbor node, and T_p is calculated using the following formula.

$$T_P = \frac{\sum a_{nb} T_{nb}{}^* + b}{a_P} \tag{8}$$

In the abovementioned equation, T_{nb}* indicates the value of neighbor node in the memory of the computer. For the neighbors considered previously during the current iteration, the value of T_{nb}* is related to the previous iteration. Anyhow, T_{nb}* is the last accessible value for the temperature of neighbor node. After all nodes are taken into consideration, one iteration is completed using the method of Gauss – Seidel Point-by-Point. The said method is not always convergent. Indeed, whenever Scarborough criterion (1985) is true, the convergence of Gauss-Seidel method is guaranteed . This is as follows:

$$\frac{\sum |a_{nb}|}{|a_p|}\begin{cases}\le 1 & \textit{for all equations} \\ < 1 & \textit{Atleast for one equation}\end{cases} \tag{9}$$

In case the interval times are chosen properly, the above mentioned convergence relation will be acceptable[18]. The time interval used in this research is equal to 0.01 second, which covers the convergence condition of Gauss – Seidel very well.

2.3. Case Study

2.3.1. Physical Model

The physical model of this case study is made of Plexiglas prepared at Utah Water Research Laboratory (UWRL) based on standard mold. The dimensions of this model are as follows, fig. 2 [5]:

Design head (H_d) = 0.301 m

Width = 1.83 m

Height of spillway crest (P) = 0.8127 cm

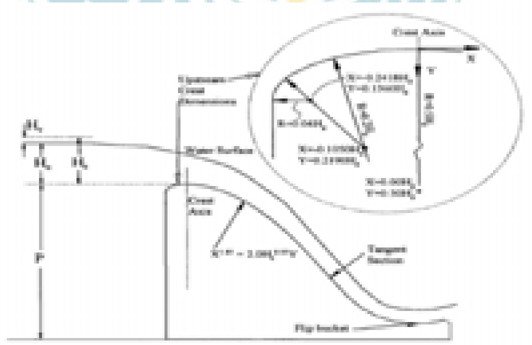

Fig. 2. The Dimensions of Ogee Spillway and Flow Parameters

2.3.2. Boundary Conditions

In general, one of the most important phases of the numerical analysis of flow field is to determine proper boundary conditions, which are matched appropriately with the physical conditions of the problem. For numerical modeling of ogee spillway, the boundary conditions of the fig. 3 have been provided.

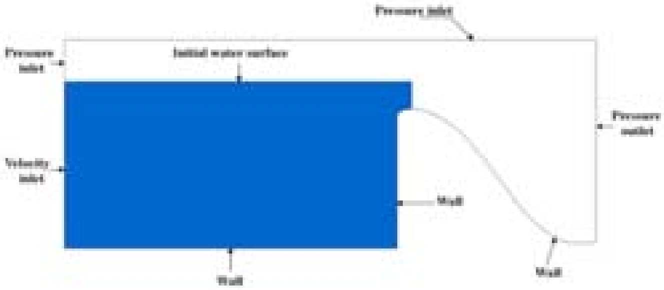

Fig. 3. The Boundary Conditions of the Ogee Spillway of Simulated Numerical Model

2.3.3. Validation of Numerical Model

In this research, it is required that the accuracy of the results obtained from numerical study to be verified. Before such verification, the validity of the selection of turbulence model, the accuracy of meshing and the fact that they have no impact on the results shall be ensured. For validating the numerical model, quad-pave meshing was used as it is shown in the fig. 4.

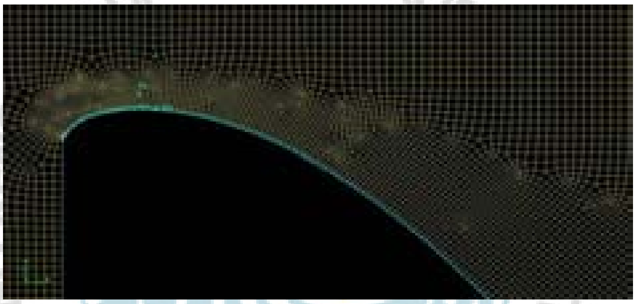

Fig. 4. Meshing and its distribution in the model of ogee spillway [21]

To have access to an appropriate turbulence model for calculating the parameters of flow over ogee spillway, numerical model with meshing of 0.0065 m was taken into consideration within 70 seconds and step time of 0.01 second [21]. By studying different models of turbulence under the same conditions, the best and worst simulated flow profile can be accessed in comparison with laboratory model, which have been shown in fig. 5 and fig. 6.

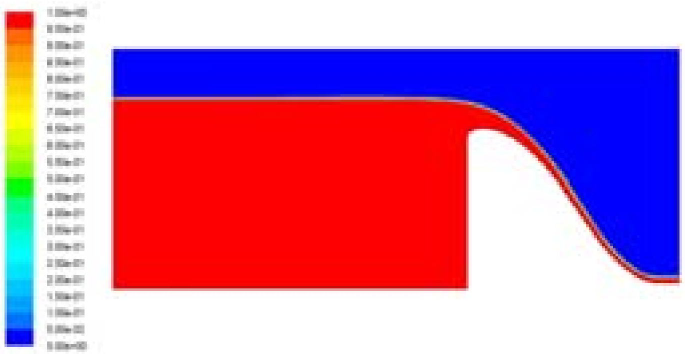

Fig. 5. Flow Profile in RNG k – ε Turbulence Model

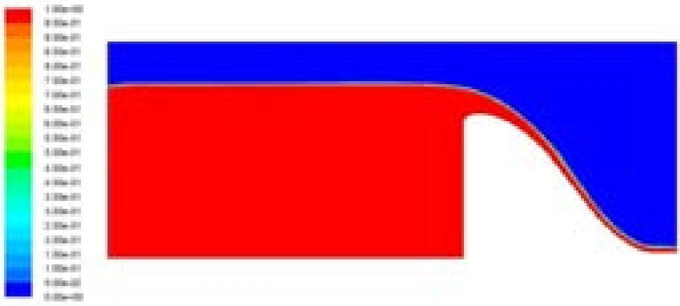

Fig. 6. Flow Profile in Standard k – ω Turbulence Model

3. Conclusion

The results obtained from the simulation of the parameters of numerical method have been compared with the laboratory measures, and based on the percent of relative error, the most appropriate choice for turbulence model suitable for the proper calculation of the parameters of flow over ogee spillway. According to the equation (6), relative error is equal to the modulus of the difference between the value of numerical method and laboratory value divided by the value of numerical method.

$$Percentage\ \ of\ \ relative\ \ error = \frac{numerical\ \ method\ \ - labratoty\ \ value}{numerical\ \ method} * 100 \tag{10}$$

The table 2 shows the numerical results obtained from discharge parameter and laboratory value for different models of turbulence.

Turbulent model	Type	Q(m^3/s)	Percentage of relative error	Profile accuracy
K- ε	Standard	0.115730309	11.438	Bad
	Realized	0.11670064	10.512	Good
	RNG	0.117578074	9.687	Excellent
K- ω	Standard	0.115765975	11.404	Poor
	SST	0.117377375	9.875	Bad

Table 2. Analysis of turbulent model

The study of the flow profile of different turbulent models based on table 2 indicates that the flow profile obtained from RNG k – ε model is more similar to laboratory model, and in contrary the standard k – ω model has the minimum similarity with laboratory model. According to the proper convergence within the step time of 0.01 second, it can be claimed that the convergence condition of Gauss – Seidel can be a proper choice for the convergence of the equation of flow over ogee spillways. The relative error of RNG model is to its minimum amount, and the profile of flow over spillway matches the physical model very well. It can be claimed that this model is an appropriate choice for modeling ogee spillways. Since the production of physical models is of high expense, and the problems due to the effects of scale can be seen in the results, therefore numerical models are recommended. It seems that RNG k – ε turbulence model is the most suitable model for the simulation of flow over ogee spillway. Thereafter, realizable k – ε model and other models specifically the standard k – ω model show higher error in modeling flow over ogee spillway. Quad-pave meshing is a proper choice for the numerical modeling of flow over ogee spillway. In case of not using proper turbulence model, the results obtained from numerical method show considerable lower relative error in comparison with laboratory results. It seems that in case the RNG k – ε turbulence model is used, the accuracy of the results obtained from the flow over ogee spillway is increased. The percentage of relative error of the flow discharge over ogee spillway is 9.687 indicating that this method is of high accuracy. Whereas the decision criterion (P-value) of SPSS software is about 0.000006, and is very lower than 0.05, therefore it can be claimed that this numerical study is of high precision.

4. Discussions

In 1980, Rodi used the standard k – ε turbulence model to analyze flow over ogee spillway, and he found that this model is highly accurate [5]. Moreover, Ho et al. (2001), Kim (2003) and Savage et al (2001) used Flow 3D software and standard k – ε turbulence model to model the profile of the flow over ogee spillway, and found that the numerical model and laboratory sample are matched very well [10]. In this research, Fluent Software and RNG k – ε turbulence model has been applied, whose accuracy is higher than the other models significantly.

The discharge rate obtained from the numerical solution of the Fluent Software has been equal to 0.1175781 m^3/s , while the discharge rate obtained by Bruce et al. using Flow 3D software is equal to 0.125208 m^3/s [5]. The USBR graphs show the discharge equal to 0.119944 m^3/s and USACE graphs show the discharge rate equal to 0.1222 m^3/s.

By considering the results of the abovementioned methods, it can be observed that the results of this research are very similar to the results of USBR graphs, and are of high precision [19]. Decision criterion (P-value) obtained from numerical study is about 0.000025, which is in comparison with the results of the study of Bruce et al. conducted using Flow 3D software, of high accuracy [5]. The decision criterion (P-value) obtained from numerical method of Fluent software with USBR graphs is approximately equal to 0.001037 and with USACE graphs is equal to 0.000138, which are lower than 0.05, and it proves the high accuracy of the numerical method used in this research [19]. Based on the abovementioned facts, and this research to analyze the sensitivity of meshing, it can be claimed that RNG k – ε turbulence model is on great precision.

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