Natural Convection – An Overlooked Phenomenon of the Solidification Process

A. Reikher, Product Design Leader
H. Gerber, Sales, Engineering & Technology Leader
Albany Chicago LLC
Pleasant Prairie, Wisconsin

K.M. Pillai, Associate Professor
T.-C. Jen, Professor
Department of Mechanical Engineering
University of Wisconsin – Milwaukee
Milwaukee, Wisconsin

Global competition, constant demands for cost reduction, pressure to shorten product development time combined with the increased complexity of products are among the many challenges that die casting companies face today. Anyone who has worked on the difficult pressure die casting projects knows the complexity of the system that forms the casting. The parameters and the interactions are complicated and difficult to predict without the aid of computing power from faster processors. This project encompasses many topics, including material factors, temperatures, vector flows, pressure, fill time, tool temperature in multiple locations, geometry challenges and solidification speeds and flows. Increasingly, we are asked to improve the physical properties of the castings. We will produce light metal solutions that are specified to supplant heavier steel components must be produced at lower cost levels. As the market becomes increasingly globalized and "commoditized," the need to improve the speed to launch with enhanced properties, and the corresponding achievement of first-shot success, is critical. These may be the only advantages local companies can build their futures upon; the ability to respond quickly with the most innovative solution, with the least risk to the OEM.

Recent developments in computer science, as well as development of the efficient numerical algorithms, have helped to establish Computational Fluid Dynamics (CFD) software as a standard tool for die casting process development. Under pressure to reduce the product development cycle and reduce computational time, engineers tend to assume a "zero velocity field" during the alloy solidification stage of the die cast process. Although this assumption holds true in thin-wall castings, convective flow can substantially change the shape and location of the solid- liquid interface as well as the location of the thermal neutral axis in thick-wall castings. In this article, numerical analyses of the alloy solidification with the effect of natural convection are presented.

Introduction

Thermal analysis plays an important role in defect management of the casting process. Management of the movement of solid-liquid interface, identification of the area of solidification, as well as determination of position and shape of thermal neutral axis, allow engineers to predict, manage and eliminate the majority of heat-related defects in castings. Recent development in computer science, as well as development of the efficient numerical algorithm, has helped to establish computational fluid dynamics software as a standard tool for die cast process development.

In order to expedite the development process it is common to assume a "zero velocity field" during solidification stage of the die casting process. In many cases the problem can be reduced to one-dimensional analysis with a known analytical solution. This assumption also allows for substantially reduced computational time of the numerical solution. For a thin-wall casting, the one-dimensional analyses with a "zero velocity field" will obtain a result with acceptable accuracy in a relatively short period of time. On the other hand, thick-wall castings require special attention due to longer solidification time and higher temperature gradient between the metal at the middle of the wall and surfaces along the steel cavity walls. This article examines how natural convection can substantially influence location and shape of the solid-liquid interface, rate of solidification and temperature distribution.

Description of the Problem

Free or natural convection can be defined as circulation of fluids caused by buoyancy from density changes induced by heating or cooling. There are significant differences between free and forced convection. In natural convection, momentum not only affects advection in the energy equation, but also, the density gradient in a gravity field, since it is in the momentum equation. It means that in the case of natural convection, the momentum equation cannot be solved independently of the energy equation.

Several attempts were made to analytically and numerically analyze solidification of the molten metal without considerations for natural convection (Stefan problem). These types of problems are usually solved using the enthalpy technique[1], which allows incorporation of both sensible and latent heats into the governing energy equation and eliminates the need to be explicitly tracking the solid-liquid interface. As shown in this case[2], the solid-liquid interface remains parallel to the cold wall.

This article compares results of the classical Stefan problem applied to a solidification process (assuming stationary metal during solidification) and solution which will include natural convection. A square-shaped cavity with two vertical walls cooled to a temperature below temperature of solidification was examined. Molten aluminum temperature was assumed to be above temperature of solidification. Governing equations and boundary conditions are presented in the Appendix

Table 1 – Properties of pure aluminum.

k $\left(\frac{J}{smK}\right)$	μ $(Pa \cdot s)$	C $\left(\frac{J}{kgK}\right)$	ρ $\left(\frac{kg}{m^3}\right)$	β $\left(\frac{1}{K}\right)$	g $\left(\frac{m}{s^2}\right)$	L (m)	T_∞ (K)	T_c (K)
96.4	0.00125	1086	2710	$2.1e^{-5}$	9.81	0.05	922	477

(available at www.diecastingengineer.org/issues/files/reikher.pdf). Properties of pure aluminum are shown in Table 1.

Here, k is the heat transfer coefficient, μ the dynamic viscosity, C the specific heat, ρ the density, β the coefficient of thermal expansion, g the acceleration due to gravity, L the characteristic length, T_∞ the temperature of the molten aluminum, and T_c the temperature of the cavity wall.

Based on material properties the coefficient of diffusivity $\alpha = \frac{k}{\rho C}$, kinematic viscosity $v = \frac{\mu}{\alpha}$,

Prandtl number $Pr = \frac{v}{\alpha}$, and Rayleigh number

$$\sqrt{\frac{g\beta L^3(T_\infty - T)}{\alpha v}}$$

can be calculated. Table 2 shows calculated parameters.

Table 2 – Calculated dimensionless variables.

α $\left(\frac{m^2}{s}\right)$	v $\left(\frac{m^2}{s}\right)$	Pr	Ra
$3.276e^{-5}$	$4.612e^{-7}$	0.014	$7.58e^{5}$

Based on a very small Pr number, we can state that maximum gravity-driven velocity occurs close to the wall and diminishes as it approaches boundary layer. The boundary layer can be simply defined as a distance from the wall where most of the temperature and velocity changes take place. The approximate thermal boundary layer thickness can be defined as Equation 1[3]:

$$\delta_t = \sqrt{\frac{\alpha}{v}}\left(\frac{v^2}{g\beta\Delta TL^3}\right)$$

Fluid outside of the boundary layer will not be affected by the convective flow due to insignificant viscous diffusion. Velocity and temperature distribution in the cavity affected by natural convection are shown in Figure 1.

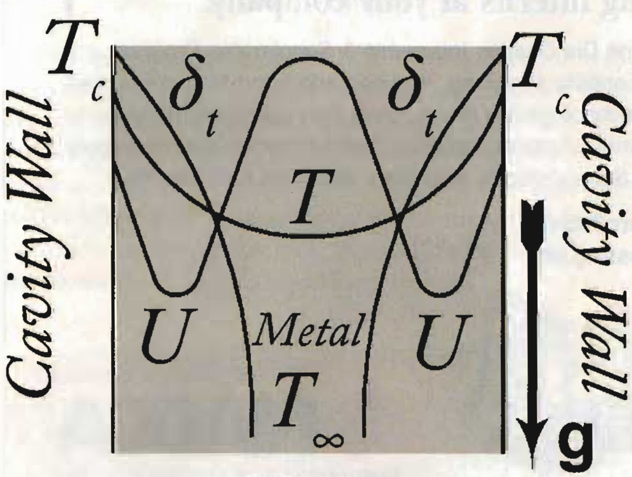

Figure 1 – Velocity U, Temperature T and Boundary layer distribution δ_t in molten metal ($T_c < T_\infty$).

Numerical Analysis

Now, this article will look at solidification of a pure aluminum. Initially at t=0s, metal at the temperature above melting occupies a rectangular cavity with left and right wall at a temperature significantly lower than the temperature of solidification. At time t>0, time-dependent solid-liquid interface is formed. It divides metal in the cavity into two domains. The software used in this study utilizes the incompressible transient Navier-Stokes equations for modeling flow in the molten metal region driven by natural convection. Flow is assumed to be laminar.

The most important quantities in heat transfer are represented by a Nusselt number. In order to ensure grid independence of the results, analyses were run with several grid densities. The result of temperature distribution of the metal was obtained in the region next to the vertical wall. Certain code was used to calculate the average Nusselt number. Results of these analyses are shown in Table 3. All analyses were conducted with Ra = $7.85e^5$. 100 X 100 and 120 X 120 grid densities differ by less than 2%. Further analyses were performed using 100 X 100 grid density.

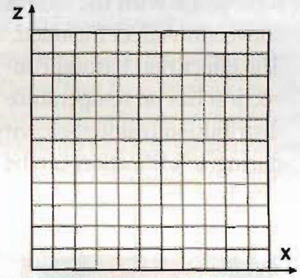

Figure 2 – Computational grid.

Table 3 – Grid independence evaluation.

Grid Density	20x20	80x80	100x100	120x120
Nu	0.61598	0.68677	0.73014	0.71752
Error(%)		11.5	6.3	1.8

Results

Numerical analyses were conducted for square dimensionless cavity 1x1 with mesh density 100x100. Figure 3 shows the basic pattern of temperature distribution with the effect of natural convection at time t>0. Analyses were run to verify temperature distribution in the cavity of the die cast die with and without effects of natural convection. In both cases,

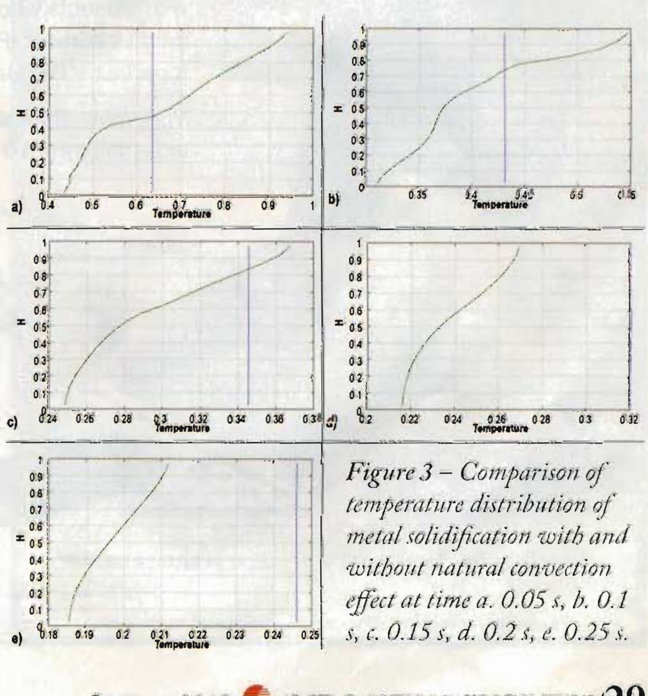

Figure 3 – Comparison of temperature distribution of metal solidification with and without natural convection effect at time a. 0.05 s, b. 0.1 s, c. 0.15 s, d. 0.2 s, e. 0.25 s.

solution was recorded every 0.05 second. The straight line represents temperature distribution without consideration of natural convection, and the curved line shows the effect that natural convection has on temperature distribution. Results of solid phase formation with the effects of natural convection are shown in Figure 4. Figure 5 shows the pattern of formation of the solid phase with the natural convection effect included. The effect that natural convection has on temperature distribution inside the cavity during solidification can be clearly seen.

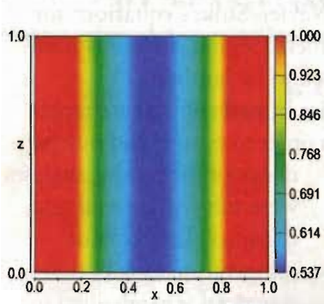

Figure 4 – Solid fraction contours of metal solidification in zero velocity field.

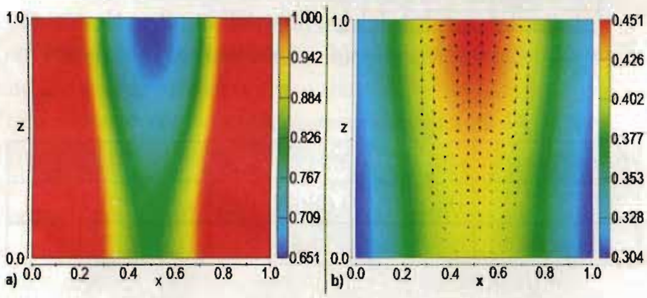

Figure 5 – Solid fraction contours of metal solidification including effects of natural convection; a) solid fraction contours, b) velocity distribution.

Next, solidification analyses were run with different aspect ratios of the cavity. As shown in Figures 6-9, as wall thickness decreases, the effect of natural convection on temperature distribution diminishes. Explanation for this effect can come from the definition of the boundary layer. As it was indicated above, gravity-driven flow in molten metal is established only inside the boundary layer. Metal outside the boundary layer is not dragged along. But since the analyses were conducted in a closed cavity, gravity-driven downward flow creates a returning upward stream. When the distance

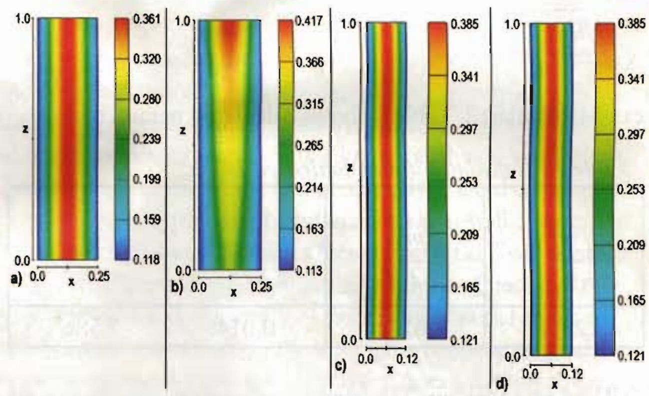

Figure 6 – a) Distance between walls 5X of boundary layer (solidification in zero velocity field); b) distance between walls 5X of boundary layer (solidification with the effects of natural convection); c) distance between walls 3X of boundary layer (solidification in zero velocity field); d) distance between walls 3X of boundary layer (solidification with the effects of natural convection).

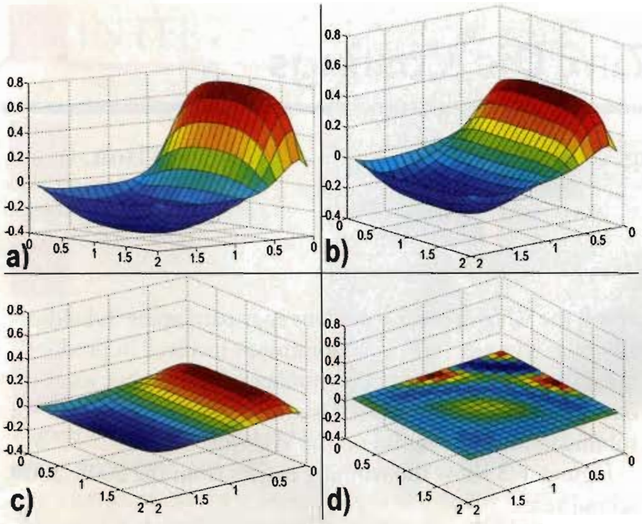

Figure 7 – a) Velocity distribution in cavity with cold walls at 90° to horizontal plane; b) velocity distribution in cavity with cold walls at 60° to horizontal plane; c) velocity distribution in cavity with cold walls at 30° to horizontal plane; d) velocity distribution in cavity with cold walls at 0° to horizontal plane.

between the walls is less than four times the boundary layer, upward flow interferes with gravity-driven flow. As a direct result, they cancel out each other.

Next, analyses were conducted to establish a relationship between cavity wall inclination and convective flow patterns. Figure 7a shows flow velocity distribution in cavity with cold walls positioned at 90° to the horizontal plane. Figures 7b, 7c and 7d show velocity distribution in the cavity with cold walls positioned at 60°, 30° and 0°, respectively, to the horizontal plane. Natural convection flow velocity as well as the Nusselt number changed substantially with the change of the cavity wall inclination angle. The highest flow velocity corresponds to the cavity walls at 90° to the horizontal plane.

Conclusion

The natural convection effect has to be accounted for in the calculation of temperature distribution, solidification time and estimation of the last-to-solidify area. If the casting wall's thickness and orientation in the cavity allows for natural convective flow to exist, analyses of solidification will yield an error in temperature distribution, time of solidification and the last area of solidification, as well as the locations of temperature related defects.

About the Authors

Alexandre Reikher is a product design leader at Albany Chicago Co. in Pleasant Prairie, WI, the company he joined in 1997. He is responsible for all aspects of product design, finite element and computational fluid dynamics analysis. He has 25 years of experience in the die casting industry. He holds five patents and co-authored the book: Casting: An Analytical Approach.

Harold Gerber (Hal) is the sales, engineering and technology leader at Albany Chicago Co. He has worked in the die casting industry since 1974, developing designed applications for machined pressure die castings. Gerber earned a bachelor's degree in organizational behavior (with math, science minors) and a master's degree in engineering management, both from Northwestern University. He is a long-standing member of NADCA and its predecessor SDCE, and he currently serves on the NADCA Board of Governors. Gerber has participated in the Process Development and the Computer Modeling Task Forces and shares a patent with ACC on casting insert technology.

Dr. Krishna M. Pillai is associate professor at University of Wisconsin – Milwaukee. He is also the director of Laboratory for Flow and Transport Studies in Porous Media at UWM. His research interests lie in several areas of porous media transport, including flow and transport in fibrous media, wicking in rigid and swelling porous media, and evaporation modeling using network and continuum models. He was awarded the prestigious CAREER grant in 2004 by the National Science Foundation of USA to model and simulate flow processes during mold filling in liquid molding processes used for manufacturing polymer composites.

Tien-Chien Jen is professor and chairman of the Mechanical Engineering Department at the University of Wisconsin – Milwaukee. He earned his Ph.D. in mechanical engineering from the University of California. He received the prestigious NSF-GOALI award in 1999 for the first of its kind that has been awarded to UWM. He also received the 2000 UWM Foundation/Graduate School Outstanding Research Award, the 2001 CEAS Outstanding Research Award, Research Initiation Awards from the Society of Manufacturing Engineers two years in a row and the 1989 Best Paper of the Year Award from International Institute for Production Engineering Research (CIRP).

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