A NUMERICAL AND EXPERIMENTAL STUDY ON THE CHARACTERISTICS OF HYDRAULIC JUMPS ON ROUGH BEDS

DENIZ VELIOGLU(1), NURAY DENLI TOKYAY(2) & ALI ERSIN DINCER(3)

(1) Department of Civil Engineering, Middle East Technical University, Ankara, Turkey, vdeniz@metu.edu.tr
(2) Department of Civil Engineering, Middle East Technical University, Ankara, Turkey, ndenli@metu.edu.tr
(3) Department of Civil Engineering, Middle East Technical University, Ankara, Turkey, aliersin@metu.edu.tr

ABSTRACT

Baffle blocks and sills are common accessory devices which are used in order to stabilize the location of a hydraulic jump and shorten the length of a stilling basin. On the other hand, strip roughness elements which cover the entire length of a basin may be an alternative. The objective of this study is to determine the effects of this type of roughness elements on the characteristics of hydraulic jumps such as conjugate depth ratio, jump length and energy dissipation. The study is carried out using experimental data and a computational fluid dynamics (CFD) model, namely Flow 3D. In the first phase of the study, the experimental data are compared with Flow 3D results in order to assess the sensitivity of the code. In the second phase, several investigations are made to determine whether strip roughness elements are effective on the characteristics of hydraulic jumps or not. The results show that strip roughness elements have positive effects on the characteristics of hydraulic jumps. The tail water depth reduction compared to classical jump is 18-20%. The length of the jump is reduced about by 20-25%. This type of roughness elements induce 2-3% more energy dissipation than that of a classical jump. Therefore, strip bed roughness elements may be considered as an alternative for baffle blocks and sills.

Keywords: Hydraulic Jump, Strip, Roughness, CFD

1. INTRODUCTION

A hydraulic jump, which was first described by Leonardo da Vinci in the 16th century, is defined by Chow (1959) as the abrupt change in the direction of flow in an open channel under certain conditions, where the flowing stream passes from supercritical state to subcritical state. Another definition given by Thompson and Kilgore (2006) is that hydraulic jump is the process where water surface moves upwards through critical depth as kinetic energy is converted to potential energy.

In order to be able to describe hydraulic jump properly, one needs to fully understand its characteristics, which are jump length and conjugate depth. Conjugate depth is defined by Carollo et al. (2007) as the depths immediately before and after the jump. Jump length is defined as the distance between the two cross-sections with the conjugate depths. Hydraulic jumps have many practical applications such as raising water level, mixing chemicals used for water purification and aerating water. The most important practical application of hydraulic jumps is to dissipate energy of supercritical flow at the foot of a spillway. For this purpose, a stilling basin must be designed. The desirable features of stilling basins are to promote the formation of the jump and to make the jump stable in one position. The most economical stilling basin design can only be achieved by keeping the basin length as short as possible. Negm (2000) states that the performance or efficiency of any stilling basin is usually assessed in terms of the characteristics of the jump it allocates. In order to achieve efficiency and economy, certain devices are used by installing them into the basins. One of these devices is roughness elements, on which this study focuses.

Rough beds affect the reduction of the jump length and tail water depth positively. To illustrate, an increase in the bed shear stress is caused by the interaction of the flow with the rough bed. This leads to an increase in energy dissipation and, as a result, a decrease in jump length, which are required for the efficiency of basin design.

In this study, an attempt is made to investigate the effects of strip roughness elements on the characteristics of hydraulic jumps both experimentally and numerically. For numerical modelling, Flow 3D code, which is rather satisfactory for free surface flow equations, is used. Comparison of the experimental data with Flow 3D results proved that the code indeed gives reliable results. The data obtained from experiments are used to determine the change in the conjugate depth ratio, jump length and energy dissipation occurring due to the introduction of strip roughness elements to the stilling basin. In short, this study is expected to contribute to the available knowledge concerning the effects of rectangular prismatic roughness elements on the main characteristics of the hydraulic jump and to support the findings of the previous investigations.

2. THEORETICAL CONSIDERATIONS

2.1 Characteristics of Hydraulic Jumps

2.1.1 Conjugate Depths

The momentum equation is applied along the flow direction of a simple hydraulic jump that occurs on a smooth rectangular channel (Figure 1) to obtain the famous Belanger equation as explained by Chow (1959):

$$\frac{y_{2s}}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^{\,2}} - 1\right) \qquad [1]$$

where,

y_1 is the depth of supercritical flow, y_{2s} is the conjugate depth of supercritical flow on smooth beds and Fr_1 is Froude number of supercritical flow which is given by:

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} \qquad [2]$$

where,

V_1 is the average velocity of supercritical flow, g is the gravitational acceleration

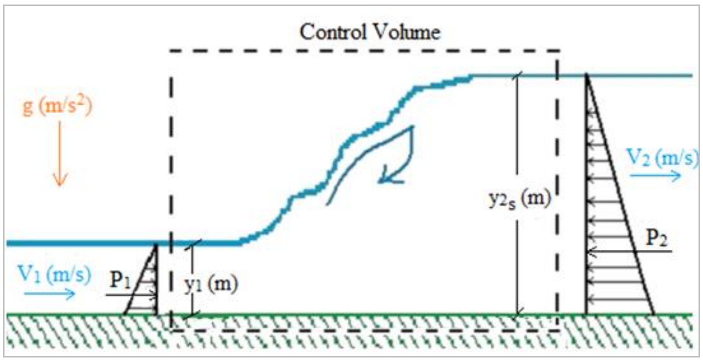

Figure 1. Application of momentum equation for a hydraulic jump (Velioglu, 2012)

2.1.2 Jump Length

The length of a hydraulic jump is an important parameter in designing stilling basins since it determines the length of the basin. However, there is no theoretical or exact formulation regarding the length of a hydraulic jump. There are some empirical relations based on experimental data. USBR (1955) developed a chart considering the relation between the length ratio and upstream Froude number shown in Figure 2.

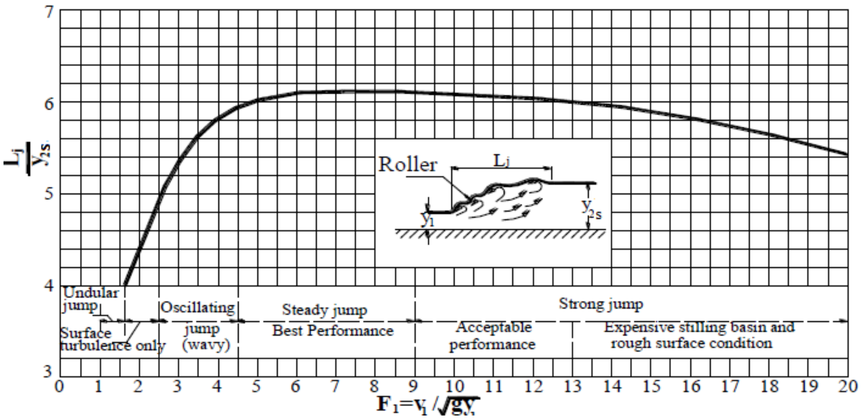

Figure 2. Length of a hydraulic jump with respect to upstream Froude number (USBR, 1955)

The related formula is given as (Chow, 1959):

$$L_j = 6.1y_{2s} \quad [3]$$

Also, the data collected by USBR (1955) is expressed in another form by Elevatorski (1959) as:

$$L_j = 6.9(y_{2s} - y_1) \quad [4]$$

2.1.3 Energy Dissipation

The amount of energy dissipated in a hydraulic jump is too large to be neglected. Therefore, hydraulic jumps are considered as one of the most effective ways of dissipating energy in water structures. The main causes of energy loss during a hydraulic jump are turbulent flow and secondary waves. When the principles of continuity, conservation of momentum and energy are applied between upstream and downstream sections of a hydraulic jump, it is possible to determine the amount of energy dissipated.

For horizontal and rectangular channels with a constant width, from the general energy equation, the energy dissipated can be obtained as:

$$E_L = E_1 - E_2 = (y_1 - y_2) + \frac{q^2}{2g}\left(\frac{y_2^{\,2} - y_1^{\,2}}{y_1^{\,2} y_2^{\,2}}\right) \quad [5]$$

where,

y_1 is the supercritical flow depth, y_2 is the conjugate depth of y_1, E_1 is the specific energy at the upstream section of the jump, E_2 is the specific energy at the downstream section of the jump, q is the discharge of flow per unit width and E_L is the amount of energy dissipated in the hydraulic jump.

2.2 Experimental Setup

Experimental study is conducted in a horizontal, rectangular open flume. The flume is 25 cm wide, 1000 cm long and 43 cm deep. The entry and outlet of the flume is made of concrete, whereas the middle section is fiberglass and 364 cm long. The roughness elements are located in the middle section and they are made of the same fiberglass material (Figure 3). A sluice gate is placed 20 cm upward from the first roughness element. It is used to produce a uniform supercritical flow with a constant depth of y_1 and it is adjusted to have the pre-determined supercritical flow depths. An adjustable weir controlling the tail water depth is placed at the end of the flume. Water coming from downstream weir is collected in a basin which is connected to a return channel having 25 cm width, 40 cm depth and 745 cm length. Discharge measurements are made by a V-notch weir with a notch angle of 30^0 which is located at the end of the return channel. The incoming flow velocity, V_1, and thus the upstream Froude number, Fr_1, are calculated using discharge passing through the system and incoming flow depth, y_1 for each experimental session.

Prismatic rectangular bars are used as roughness elements in the experiments. The crests of roughness elements are at the same level as the upstream bed. All roughness elements have a width of 25 cm and a length of 1 cm. The height of roughness elements are 1 cm. The longitudinal distance between two roughness elements are taken as 4 cm.

Experiments are conducted both on smooth bed and rough beds having rectangular prismatic bars in strip form. Froude number of the incoming flow varies between 6.8 and 16.6. Detailed information on the experiments can be found in Evcimen (2005).

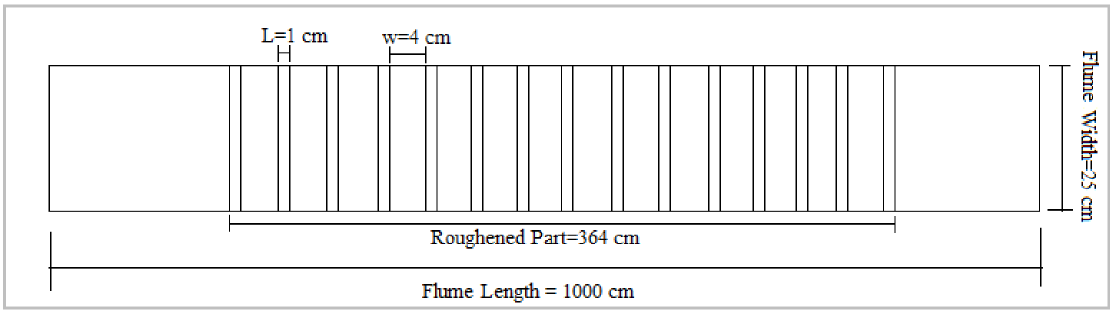

Figure 3. Plan view of the experimental flume (not to scale)

2.3 Flow 3D Model

Flow 3D is a computational fluid dynamics (CFD) model which uses volume of fluid (VOF) method. It is possible to simulate discontinuities in the flow (i.e. hydraulic jumps) via this method. Flow 3D code gives satisfactory results on free surface flows by solving the Reynolds-averaged Navier-Stokes (RANS) equations over the computational domain.

Air entrainment and turbulence are the most crucial phenomena accompanying the formation of a hydraulic jump. Therefore, both of them are included in the computations. In the air entrainment model, entrainment rate coefficient is taken as 0.5. To include effects of turbulent flow, RNG turbulence model is activated. RNG turbulence model is very similar to standard k-ε model except for some refinements. The model includes an additional term in its ε-equation which significantly improves the accuracy. Moreover, RNG model provides an analytical formula for Prandtl numbers and an analytically derived differential formula for low Reynolds numbers. In addition to these advantages, for the hydraulic jumps along corrugated beds, RNG model gives better results than standard k-ε model according to Kaheh (2010).

For x-min and x-max boundaries, specified velocity and outflow boundary conditions are applied, respectively. Velocity boundary condition allows user to work with a specified velocity as well as a defined free surface elevation at the boundary. Outflow boundary condition applies a Sommerfeld radiation condition so that the conditions at the boundary can be estimated dynamically. Symmetry boundary condition which applies a zero-gradient and a zero velocity condition normal to the boundary is used for y-min, y-max and z-max boundaries. For z-min, wall boundary condition is used. Wall boundary condition applies no slip condition between the fluid and solid surfaces as well as zero velocity condition normal to the boundary. The pressure is zero on the free surface. Pressure distribution is accepted as hydrostatic throughout the computational domain.

The dimensions of channel bed and roughness elements used in the experiments are kept intact in the model. Instead of the adjustable tailgate that is used in the experiments, a weir is introduced just before x-max boundary (Figure 4). Several gauge points are placed with 5 cm spacing along the bed to accurately locate the conjugate depths of hydraulic jump.

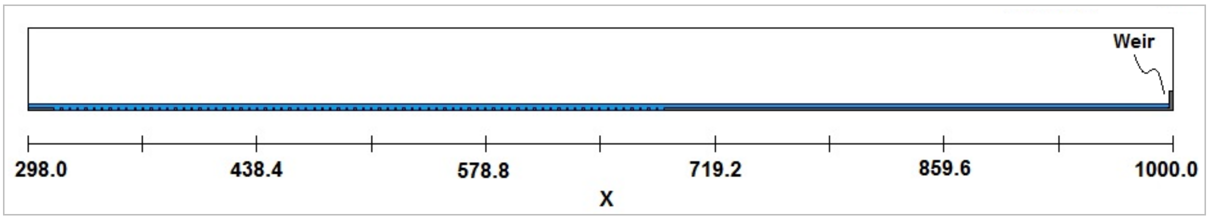

Figure 4. Model which is constructed via Flow 3D for the simulations (not to scale)

Upstream flow depth, y_1, and Froude number, Fr_1, which are used in the experiments are taken as the input data. Simulation duration ranges from 150 sec to 200 sec, which is enough to obtain a stable jump. The profile of a hydraulic jump for y_1=1.07 cm and Fr_1=12.02 is given on Figures 5 and 6. Mesh resolution is 0.1 cm.

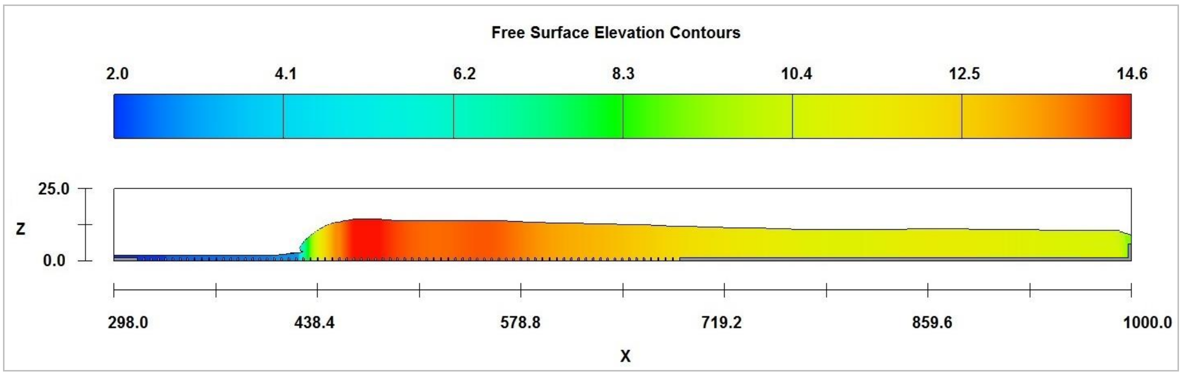

Figure 5. Free surface elevation profile of a hydraulic jump modelled by Flow 3D at t=100 sec

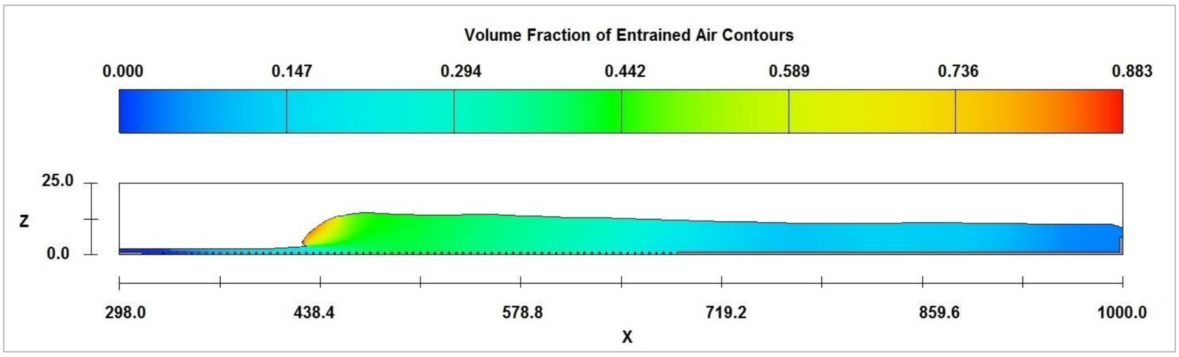

Figure 6. Volume fraction of entrained air of a hydraulic jump modelled by Flow 3D at t=100 sec

2.4 Hydraulic Jump on Rough Beds

Roughness elements are used to stabilize a hydraulic jump, to shorten the length of a stilling basin and to increase the energy dissipation on a channel bed. Baffle blocks and sills are the most common energy dissipation devices used on stilling basins (Figure 7). However, this study recommends strip rectangular bars as roughness elements which are explained in detail in Section 2.2.

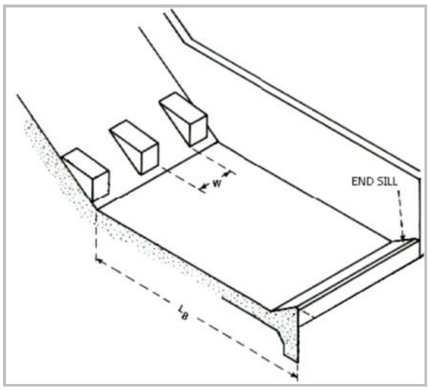

Figure 7. A stilling basin with baffle Blocks and end sill

When a hydraulic jump takes place on a rough bed, the concept of bed shear stress comes into picture. It is possible to neglect the boundary channel resistance on smooth channels because it is rather small compared to the other forces. However, if the channel bed is rough, the effect of the boundary resistance should be included in the momentum equation. To include this effect, a coefficient, β, is introduced to the momentum equation to modify the classical hydraulic jump relation for rough beds. In the scope of this Carollo and Ferro (2004b) developed an expression for the bed shear force as:

$$F_s = \beta\,(M_1 - M_2) \qquad [6]$$

where,

F_s is the bed shear force, β is a coefficient assuming values larger than zero and smaller than one, M_1 is the momentum flux at the upstream section of the hydraulic jump and M_2 is the momentum flux at the downstream section of the hydraulic jump.

The concept of hydraulic jump on rough beds is further studied by Carollo et al. (2007) and the bed shear force is included in the momentum equation of a hydraulic jump:

$$\Pi_1 + M_1 = \Pi_2 + M_2 + \beta\,(M_1 - M_2) \qquad [7]$$

where,

Π_1 is the hydrostatic force at the upstream section of the hydraulic jump and Π_2 is the hydrostatic force at the downstream section of the hydraulic jump.

After several modifications, an equation is obtained to calculate conjugate depth ratio of hydraulic jumps on rough beds:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8(1-\beta)Fr_1^{\,2}} - 1\right) \qquad [8]$$

Coefficient β is equal to zero when the bed is accepted as smooth. In other words, Equation 8 becomes exactly the same as Belanger equation. Therefore, β is the parameter which may assume different values for different types of roughness elements. This study considers only strip roughness elements, the analyses for other type of roughness elements can be found in the studies of Velioglu (2012).

3. RESULTS AND DISCUSSION

In the first phase of the analyses, reliability of Flow 3D code is assessed. As mentioned in Section 2.2, the experiments are also carried out on a smooth bed. Therefore, a separate model without roughness elements is prepared for Flow 3D simulations. The cell size used in the model is 0.5 cm. In order to test the soundness of the experimental and numerical data, the conjugate depth of each y_1 on smooth bed is compared with the ones obtained via Belanger equation. The deviation of experimental and numerical results from the analytical solution is also computed. It is seen that % error for both experimental and numerical results are in acceptable limits. For lower mesh resolutions, it is possible to obtain better results with the numerical model. The data are shown in Table 1 and Figure 8.

Table 1. Analytical, experimental and numerical conjugate depth values on smooth bed

	Q (LT/S)	Y_1 (CM)	FR_1	V_1 (CM/S)	Y_{2S-ANA} (CM)	Y_{2S-EXP} (CM)	$\%DIF_{ANA-EXP}$	Y_{2S-NUM} (CM)	$\%DIF_{ANA-NUM}$
A	11.97	1.70	6.90	281.65	15.75	13.56	13.92	14.50	6.87
B	11.97	1.68	7.02	285.00	15.86	14.58	8.07	15.00	4.28
C	13.35	1.70	7.69	314.12	17.66	15.22	13.83	16.50	5.39
D	13.35	1.68	7.83	317.86	17.78	16.23	8.72	16.50	6.09
E	14.40	1.70	8.30	338.82	19.12	15.65	18.13	18.50	2.01
F	14.40	1.69	8.37	340.83	19.18	17.21	10.27	18.50	2.32
G	16.14	1.72	9.14	375.35	21.38	18.43	13.81	20.00	5.30
H	16.14	1.67	9.55	386.59	21.74	22.48	3.41	21.00	2.23
I	16.74	1.71	9.56	391.58	22.28	21.45	3.73	21.00	4.59
J	16.74	1.70	9.65	393.88	22.35	22.54	0.83	21.00	4.89
K	13.50	1.30	11.63	415.38	20.74	18.12	12.65	19.00	7.23
L	13.50	1.29	11.77	418.60	20.83	20.67	0.78	19.00	7.68
M	15.12	1.30	13.03	465.23	23.31	21.22	8.96	21.00	8.77
N	15.12	1.28	13.33	472.50	23.51	23.38	0.53	21.50	7.45
O	16.13	1.30	13.90	496.31	24.91	22.98	7.74	23.00	6.50
P	16.13	1.30	13.90	496.31	24.91	24.43	1.92	23.00	6.50
Q	16.94	1.32	14.27	513.33	25.98	23.18	10.77	24.50	4.56
R	16.94	1.29	14.77	525.27	26.30	25.95	1.33	25.50	1.85
S	18.25	1.32	15.37	553.03	28.04	24.32	13.26	27.50	0.72
T	18.25	1.29	15.91	565.89	28.38	28.21	0.61	28.00	0.14

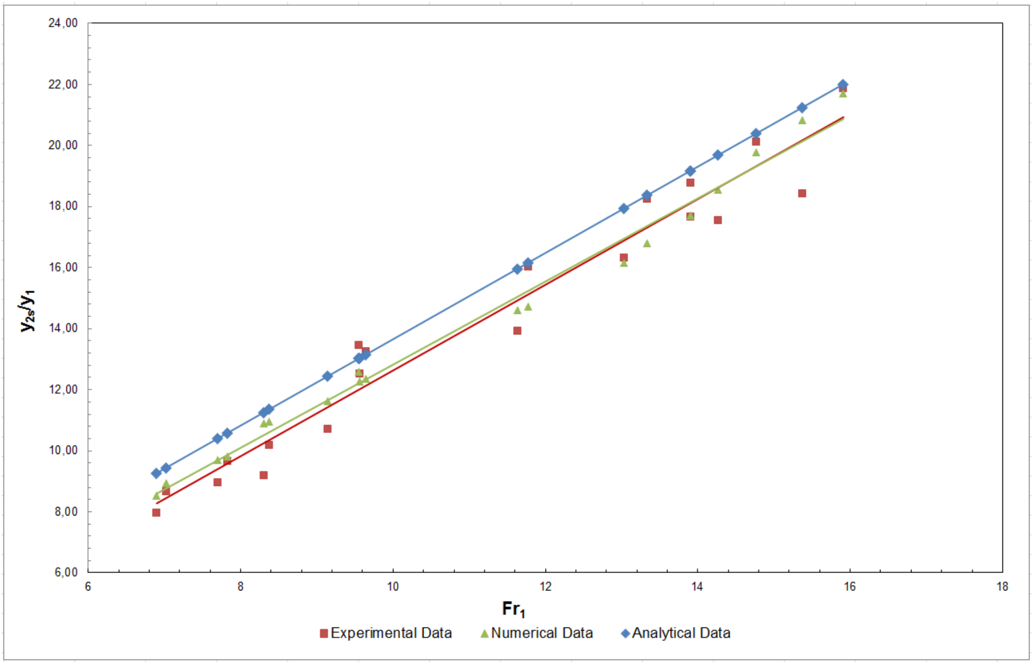

Figure 8. Analytical, experimental and numerical comparison of conjugate depth ratios on smooth bed

Another channel bed with strip roughness elements is modeled in Flow 3D to assess the capability of the code when roughness elements are present on a stilling basin. The mesh resolution of this model is taken as 0.3 in order to reach more accurate results around the roughness elements. The conjugate depth of each y_1 and related jump lengths obtained in the model are compared with the experimental results and given in Table 2.

Table 2. Experimental and numerical conjugate depth values on rough bed

	Q (LT/S)	Y_1 (CM)	V_1 (CM/S)	FR_1	Y_{2EXP} (CM)	Y_{2NUM} (CM)	%DIF	L_{JEXP} (CM)	L_{JNUM} (CM)	%DIF
A	10.52	1.11	379.10	11.49	14.75	13.90	5.76	69	66	4.35
B	10.42	1.07	389.53	12.02	15.20	14.60	3.95	61	60	1.64
C	11.70	1.08	433.33	13.31	17.08	15.90	6.91	69	72	4.35
D	11.70	1.08	433.33	13.31	16.60	15.30	7.83	78	72	7.69
E	13.60	1.33	409.02	11.32	17.79	16.90	5.00	75	74	1.33
F	15.00	1.35	444.44	12.21	19.00	17.60	7.37	76	70	7.89
G	16.24	1.38	470.72	12.79	20.06	19.30	3.79	79	75	5.06
H	12.35	1.37	360.58	9.84	14.87	14.60	1.82	61	56	8.20
I	10.52	1.34	314.03	8.66	14.02	13.90	0.86	54	50	7.41
J	13.78	1.37	402.34	10.97	17.43	16.60	4.76	74	72	2.70

The deviation between experimental and numerical results on smooth and rough beds is tolerable; therefore, Flow 3D is accepted as a satisfactory code to model hydraulic jumps even in the presence of roughness elements. However, while calculating coefficients related to hydraulic jump characteristics, experimental values are preferred in the rest of the study to reflect the real life data as much as possible.

In the second phase, the effect of strip roughness elements on the characteristics of a hydraulic jump is investigated. In order to calculate the conjugate depth ratio on rough beds, coefficient β values are determined. To determine the β values, Equation 8 can be re-arranged as:

$$\frac{y_2}{y_1} + \left(\frac{y_2}{y_1}\right)^2 = 2(1-\beta)Fr_1^2 \qquad [9]$$

and, thus:

$$Y=2(1-\beta)X \qquad [10]$$

When a graph of Y versus X is plotted, the slope of the best-fit line gives the value of 2(1-β) (Figure 9). Then, it is possible to estimate coefficient β. The slope is determined by adopting best-fit line method with the coefficient of determination, R^2, which is a statistical measure of how well a regression line reflects the real data.

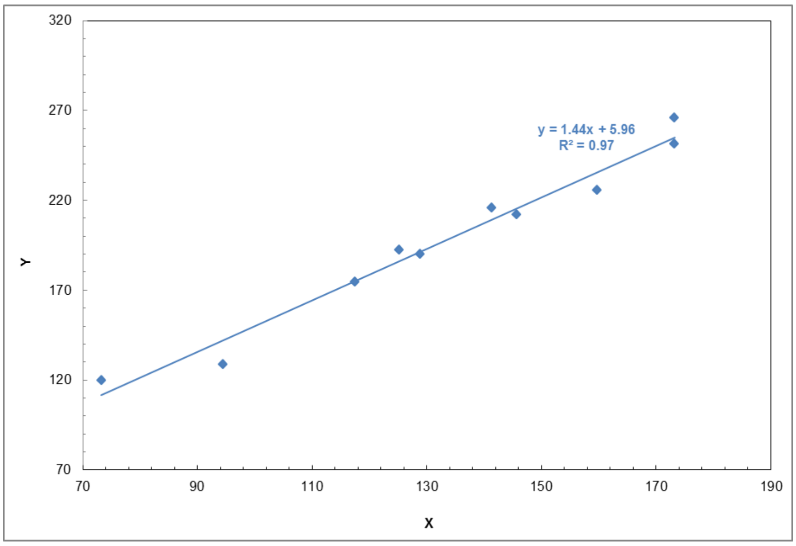

Figure 9. 2(1-β) value for rectangular prismatic bars in strip form

The best fit line and the corresponding coefficient of determination are:

$$Y=1.44X \quad \text{with} \quad R^2=0.97 \qquad [11]$$

β value is calculated as 0.28 from the slope of the best line. When this value is introduced to Equation 8, it is seen that subcritical conjugate depth values become 18-20% smaller than the ones on smooth bed. In short, strip roughness elements provide considerable reduction in the subcritical conjugate depth, y_2.

Carollo et al. (2007), Pietrkowski (1932) and Smetana (1937) observed that the roller length, L_r, is directly proportional to the difference between conjugate depths and regarding this a relationship is suggested:

$$\frac{L_r}{y_1} = a_0\left(\frac{y_2}{y_1} - 1\right) \qquad [12]$$

where,

a_0 is a coefficient regarding the relationship between L_r, y_1 and y_2.

Similarly, the same relationship may be taken as a basis for the hydraulic jump length, L_j, as follows:

$$\frac{L_j}{y_1} = \alpha\left(\frac{y_2}{y_1} - 1\right) \qquad [13]$$

where,

α is a coefficient regarding the relationship between L_j, y_1 and y_2.

The slope of the best-fit line on Figure 10 gives the value of coefficient α. It is determined as 5.74 on smooth bed and 4.67 on rough bed.

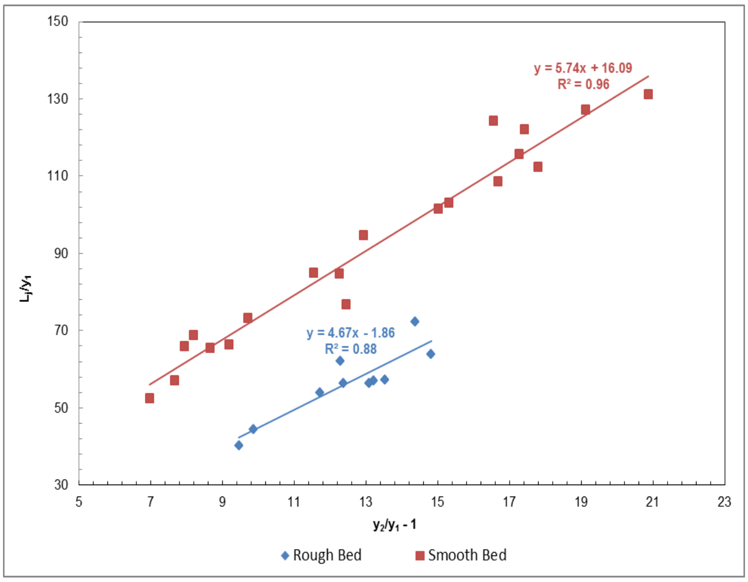

Figure 10. α value for smooth and rough beds with strip roughness elements

The reduction in jump length, R_L, may be computed as:

$$R_L = \left| \frac{\alpha_{smooth} - \alpha_{rough}}{\alpha_{smooth}} \right| x100 \qquad [14]$$

According to Equation 14, the reduction on jump length, R_L, is 18.6% when strip roughness elements are introduced to the channel bed.

It is known that there is a huge amount of dissipated energy during a hydraulic jump. Actually, this phenomenon is very useful since it facilitates economical designs of hydraulic structures. The energy loss on rough beds, h_{LR}, during a hydraulic jump can be related to the energy loss on smooth beds by:

$$h_{LR} = C_E . h_{LS} \qquad [15]$$

where,

C_E is a coefficient showing the increase in the amount of energies dissipated on rough beds, and it is greater than 1.0 if the amount of energy dissipated on a rough bed is larger than that on a smooth bed.

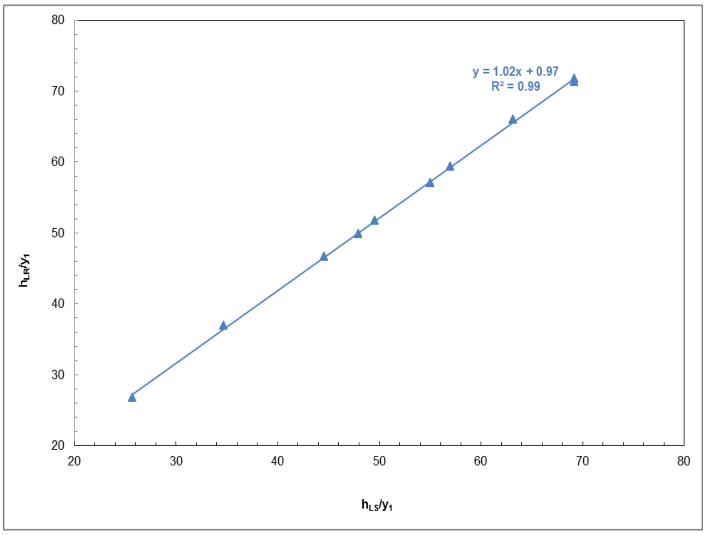

Figure 11. Coefficient C_E value

Coefficient C_E is calculated as 1.02 with R^2 = 0.99, indicating that, there is a 2% gain in the amount of dissipated energy when rectangular strip bars are used as roughness elements.

4. CONCLUSIONS

This study suggests that rectangular strip bars are used as an alternative method to stabilize the location of a hydraulic jump and shorten the length of a stilling basin. Experimental and numerical investigations both show that when strip rectangular bars are introduced to the channel bed, they have a positive effect on the characteristics of a hydraulic jump. The tail water depth reduction compared to classical jump is 19% and jump length is reduced by 20%. These roughness elements induce 2% more energy dissipation than that of classical jump. It is clear that, if height of the strip bars is increased, the reduction in tail water depth and jump length will be much more than the ones that are obtained in this study. Similarly the gain in the amount of dissipated energy will increase. The reliability of a CFD model, Flow 3D, is also tested and it is seen that Flow 3D gives satisfactory results regarding free surface flows and turbulence models. In future studies, different arrangements of rectangular roughness elements may be investigated. It might also be useful to study the effects of roughness elements on hydraulic jumps formed in trapezoidal channels.

REFERENCES

Amorim, J. C. C., Rodrigues, R. C. and Marques M. G. (2004). "A Numerical and Experimental Study of Hydraulic Jump Stilling Basin." *Advances in Hydroscience and Engineering*, Vol. IV.

Carollo, F. G., Ferro, V., and Pampalone, V. (2007). "Hydraulic Jumps on Rough Beds." *Journal of Hydraulic Engineering*, Vol. 133, No. 9, pp. 989-999.

Chow, V. T. (1959). *Open Channel Hydraulics*. McGraw-Hill, New York.

Elevatorski, E. A. (1959). *Hydraulic Energy Dissipators*. Mcgraw-Hill, New York.

Evcimen, T. U. (2005). *The Effect of Prismatic Roughness Elements on Hydraulic Jump*. M. Sc. Thesis, Middle East Technical University, Department of Civil Engineering, Ankara, Turkey.

M. Kaheh, S. M. Kashefipour, and A. Dehghani. (2010). "Comparison of k-ε and RNG k-ε Turbulent Models for Estimation of Velocity Profiles along the Hydraulic Jump*." 6th International Symposium on Environmental Hydraulics*, Athens, Greece.

Negm, A-A M. (2000). "Hydraulic Performance of Rectangular and Radial Stilling Basins." *4th International Conference on Hydro-Science and Engineering*, *Korea Water Resources Association*, Korea.

Pietrkowski, J. (1932). "Beitrag Zur Kenntnis des Wechselsprunges (Contibution on the Knowledge on Hydraulic Jumps)." *Die Wasserwirtschaft,* Nr. 25, pp. 356-358, and Nr. 26, pp. 370-372.

Smetana, J. (1937). "Studi Sperimantali sul Salto di Bidone Libero e Annegato (Experimental Studies on the Free and Submerged Hydraulic Jumps)." *L'energia Elettrica*, 24(10), Vol. 14, pp. 829-835.

Thompson, P. L., and Kilgore, R. T. (2006). *Hydraulic Jump*. Hydraulic Engineering Series No. 14, 3rd. Ed. - Hydraulic Design of Energy Dissipators for Culverts and Channels, Federal Highway Administration, Washington, D. C., 6-1 to 6-14.

USBR. (1955). "Research Studies on Stilling Basins." *Energy Dissipators and Associated Appurtenances, US Bureau of Reclamation*, Hydraulic Laboratory Report No. HYD-399.

Velioglu, D. (2012). *Effects of Different Bed Roughnesses on the Characteristics of Hydraulic Jumps*. M. Sc. Thesis, Middle East Technical University, Department of Civil Engineering, Ankara, Turkey.